The Big Beam Theory

PCI Big Beam Competition Final Proposal
Final Report

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1.0 Introduction

The Precast/Prestressed Concrete Institute (PCI) holds the Big Beam competition every year, encouraging young minds to think of innovative designs and challenge themselves in a way that their university curriculum may not. This competition has taken place annually since 2005 and up to 20 different teams are awarded prizes each year.

This year, each team is to design a 20-foot-long prestressed concrete beam that must be flat and horizontal along the top. The beam will be designed for the dead load and two asymmetrically applied loads. The location of the asymmetrically applied loads can be seen in Appendix A. The beam must not crack until at least 20 kips of pressure have been applied, and it must fail between 32 and 40 kips of pressure being applied, otherwise there are associated point reductions laid out in the competition rules. Bearing pads/plates, that do not exceed 6 inches in length, can be used at supports and/or under loads. The load may be measured at each point or the total load applied to the spreader beam may be measured. The total applied load is the sum of two-point loads. The midspan deflection must be measured as well. The beam must resist load through flexure. The beam must be consisted primarily of concrete. Longitudinal tension reinforcing shall be pretensioned and/or post-tensioned. Non-prestressed or prestressed top steel is allowed. Mesh and bar can be used for shear reinforcement. Reinforcement must be completely embedded in the beam and meet applicable spacing and cover requirements. All entries from US schools must meet the provisions of ACI-318-14 or 19 or the 8th edition of the PCI Design Handbook for a precast/prestressed beam, interior exposure.

The judging criteria will be judged in relationship to other entries in the country. Judging is based off of seven different criteria. The first criteria are design accuracy. The design accuracy for this competition is that the beam must carry at least a total factored live load of 32 kip and must not have a total peak applied load of more than 40 kip. The beam shall not crack under the total applied service load of 20 kip. Total applied load is defined as the sum of the two applied point loads. Beams meeting these criteria receive 20 points. (Beams which do NOT hold a total applied load of 32 kip shall be penalized 2 points for each kip, or part of a kip, below 32. Beams which hold a total applied load of more than 40 kip shall be penalized 1 point for each kip, or part of a kip, above 40. Beams which crack before a total applied load of 20 kip receive a 5-point penalty. The load/midspan deflection graph must show a peak load either by post-peak softening or by collapse of the beam. Stopping the test to avoid the overstrength penalty will result in a score of 0 for this category). The next criteria are lowest cost, lowest weight, and largest measured deflection at maximum total applied load. These will be judged off of the values of the best and worst performance in each category. In these criteria the points will be awarded using equation 1. These equation uses the value the team gets, the best and worse value to scale the points. The fifth criteria are the most accurate prediction of maximum total applied load, total applied cracking load and midspan deflection at maximum total applied load. In this category entries receive points based on the following scale: <10% = 10 points; deduct 1 point for each 10% increment above 10% rounded UP to the nearest 10%. Above 110% receives 0 points. Report quality is a criterion as well. Reports have to contain a discussion of the concrete mix design and the beam structural design. Judges will award 0-5 points for the quality of the report. The final criteria are practicality, innovation, and conformance with code. The judges will award

0–5 points for practicality, innovation, compliance with the applicable code, and demonstration of good engineering judgment. For any category, no entry can receive less than 0 points.

Equation 1. Point Values

$$Points = 10 * \frac{(Value\ in\ entry - Worst\ value)}{(Best\ value - Worst\ value)}$$

2.0 Technical Sections

This section includes the technical considerations the Big Beam Theory must consider when designing the beam.

2.1 Preliminary Research

All team members initially began research on their own to see what sources could be found and what information could be used. The team then got together comparing notes and identifying what areas needed more research. It was determined that three major areas of research were necessary for success in this project: three stages of design of a prestressed concrete beam, preliminary design, and the decision matrix. In the process of preliminary design of beams, the team made assumptions for dimensions, numbers of strands and steel reinforcement based on research done for the loading criteria acting on the beam. Then the calculations were carried on starting from the ultimate strength to release and to cracking respectively. In each of the following sections, it was made sure that the stresses were not exceeded and the dimensions/area of the steel works perfectly for the beam.

2.1.1 Three Stages of Design Prestressed Concrete Beam

The three stages of designing a prestressed concrete beam are release of strand, cracking load, and ultimate strength.

2.1.1.1 Release of Strand

The release of strands in the prestressed beam is an important consideration because they are in tension when the beam is poured and begins to cure. The release occurs when the part of the strand that extends beyond the concrete is cut. This causes the tension of the strand from being pulled to turn the concrete into compression when the strands are cut. This then makes the beam to camber. At this point the stress in the concrete will be approximately 174ksi. Equation 2 is used to calculate the stress due to the release of the strands. The stress at release is calculated both for the top and bottom of the beam. The equation is used for all beam types, but different values should be used for stress because of the difference in the values. The equation for calculating release at stress consists of stress due to prestressing, stress due to dead load of the beam and the bending stress due to the eccentricity. The stress at release depends on the Prestressing force (P), transformed cross sectional area

(A), transformed moment of Inertia of the beam (I) and the moment caused due to prestressing by the eccentricity (M_{PS}).

Equation 2. Release of Strand

$$\sigma_{tp} = \frac{P}{A} + \frac{M_{PS} * c}{I}$$

2.1.1.2 Cracking Load

The cracking load was calculated to ensure that the designed beam would not crack before 20 kips of pressure was applied so the team would not receive a point deduction. At this cracking point the stress in the concrete will be approximately 250ksi. Bending moment due to externally applied dead and live loads counts for the cracking moment. Cracks develop when the bottom fiber of the beam is in tension. Equation 3 is used to calculate the stress of the cracking load. The stress when the beam starts to crack depends on the Prestressing force (P), transformed cross sectional area (A), transformed moment of Inertia of the beam (I), the moment caused due to prestressing by the eccentricity (M_{PS}), the moment caused due to dead load (M_D), and the moment caused due to live load (M_{LL}). After the beam experiences a greater load than the cracking load the beam begins to crack at the bottom. The design of an appropriate section modulus will help to maximize the moment. The values of fc for light weight concrete and normal weight concrete that TPAC will supply the team with is 8000psi and 8500psi at 28 days of curing.

Equation 3. Cracking Load

$$\sigma_{crack} = 7.5\sqrt{fc_{28}} = \frac{P}{A} + \frac{M_{PS} * c}{I} + \frac{M_D * c}{I} + \frac{M_{LL} * c}{I}$$

2.1.1.3 Ultimate Strength

The ultimate strength was important to be calculated to ensure that the beam would break between 32 and 40 kips of loading as per competition guidelines. When the beam fails, the stress in the concrete will be around 265-270ksi. The flexural theory of concrete is used in determining the ultimate strength. This is the total load capacity that the beam can handle before it starts cracking. Equation 4 is used to calculate the moment of inertia of the ultimate strength of the beam. The moment of inertia when the beam starts to break depends on the cross-sectional area (A_P), tensile stress of strand (f_P), depth of the beam (d), = factor relating depth of equivalent rectangular compressive stress block to neutral axis depth (β_1), and the moment caused due to live load (M_{LL}). The teams process of calculating the ultimate strength of the beam to be between 32 and 40 kips was using different amounts of prestress strands and steel reinforcement. Trial and error are used throughout the process to obtain a realistic value to meet the moment of inertia at ultimate.

Equation 4. Ultimate Strength

$$m = A_p * f_p\left(d - \frac{\beta_1 * c}{2}\right) + M_{LL}$$

2.1.2 Preliminary Design

The three beams considered by the team are I-beam, T-beam and box.

2.1.2.1 I-Beam

The Big Beam Theory looked over past reports and observed that majority of the winning teams had an I-beam. For the MathCAD sheet used by the team an I-beam is broken down into five different areas. Appendix B shows an example of an I-beam into different areas. The triangular part of the flanges is analyzed as two areas even though there are four triangles. Out of the three selection of beams, I-beams are the hardest to form.

2.1.2.2 T-Beam

T-beams have four different areas in the MathCAD sheet. Appendix C shows an example of a T-Beam. The figure shows the four different areas. T-beams typically have a low cracking load and high ultimate load. A problem with T-beams is the balancing of the beam when breaking it. The team has noted this and added the triangular part to the bottom flanges to form stability.

2.1.2.3 Box Beam

For the MathCAD sheet used by the team a box beam is broken down into three different areas. Appendix D is a figure of a box beam. It has the breakdown of the three different areas. For box beams, the cracking and ultimate loads are within 3-4 kips of each other. They are the simplest to construct out of all the beams considered.

2.1.3 Preliminary Decision Matrix

The Precast/Prestressed Concrete Institute will consider cost, weight, and deflection when scoring the beam. The team decided to make these criteria apart of the decision matrix.

2.1.3.1 Cost

Cost is to be calculated based off of the criteria PCI provides. The criteria are based on size of strands used in the prestressed beam, type, and amount of concrete used in the beam, compression steel at the top of the beam, and formwork. Appendix E shows the tables used for determining the price of each component of the beam. The Big Beam Theory plans to use a 0.5-inch strand to tension the beam at the bottom which would cost $0.42 per foot of strand used. For the cost of concrete the cost per cubic yard will be determined by multiplying the concrete strength in ksi by $10, rounding the concrete strength down to the nearest ksi. The cost of reinforcing steel varies but is priced out by pound, costing anywhere between $0.45 and $0.70 per pound utilized. All formwork is priced at $1.25 per square yard, including all contact surfaces. All costs are based on the actual strength of the beam, not the design strength.

2.1.3.2 Weight

The beam weight will be approximated by using the measured unit weight calculated in the MathCAD sheet. This estimate must be based on the gross cross section consisting of concrete, reinforcement not included. One category of scoring is having a low weight beam; The Big Beam Theory is attempting to succeed in this category by utilizing a lighter weight concrete provided by TPAC with a unit weight of 124.1 pounds per cubic foot.

2.1.3.3 Deflection

The largest measured deflection at the maximum total applied load is another category of scoring. Correctly calculating the ultimate strength is important to have a large deflection. Deflection is calculated by using equation 5. As seen in the equation, the moment of inertia (I) is in the denominator. The team's objective is to make the moment of inertia the smallest possible value to increase the deflection.

Equation 5. Deflection

$$\delta = \frac{P * l}{E * I}$$

2.2 Preliminary Beam Design

Preliminary design was challenging for the team initially, as prestressed concrete is not covered in the Northern Arizona University Civil Engineering curriculum and the team needed guidance to ensure the process was done correctly.

2.2.1 Initial Beam Design

Initial beam design was started in November 2019 when the team prepared a MathCAD sheet that calculated the beams majors' characteristics such as cracking load and ultimate strength. This MathCAD sheet was then used to come up with multiple designs the team could potentially use for the competition. The team has come up with six different beams. There is a T-Beam, I-Beams, and box beam. Each of these designs have both a normal weight concrete and a lightweight concrete as shown below in the decision matrix. Appendix F shows the MathCAD sheet the team used. The team input the givens and the equations needed and used the MathCAD sheet to calculate unknowns.

2.2.2 Decision Matrix

The decision matrix that was used by The Big Beam Theory is located in Appendix L

2.2.2.1 Mix Selection

Two concrete mix options were made available to the team to by TPAC; a lightweight mix and a normal weight mix. The Big Beam Theory chose to use the lightweight option TPAC offered because it is still considered to be a normal weight concrete by the rules of the PCI Big Beam competition. Because of this, no penalty to the team score would be issued and this would allow for a lighter beam without being too light. In the PCI competitions they dock points for being an actual lightweight mix; luckily the "lightweight" mix from TPAC is not classified as a lightweight mix for the competition.

2.2.2.2 Beam Selection

Beam selection will consist of choosing the aspects of the preliminary designs that have received the highest score in each category: cost, weight and deflection. The lowest possible cost and weight of the beam is the goal, while achieving the largest deflection before the beam fails. Based off of the decision matrix below the I-Beam is the best beam the team came up with.

2.2.2.3 Cost

The cost of the beam uses Appendix E to calculate the cost for each beam. Concrete, strands, steel, and forming are all considered when calculating the cost of a beam. The calculations for cost can be seen in Appendix G. As seen in the Appendix G, the highest cost is the hollowed box and the lowest cost is the T-beam.

2.2.2.4 Weight

The weight of the beam uses the unit weight of reinforced concrete and the volume of the beam. The calculations for weight can be seen in Appendix G. As seen in Appendix G, the lowest weight is the I-beam and the highest weight is the hollowed box.

2.2.2.5 Deflection

Deflection is calculated by using equation 5. As seen in the equation, the moment of inertia (I) is in the denominator. The team's objective is to make the moment of inertia the smallest possible value to increase the deflection. Per the criteria of the competition a greater deflection is desired.

2.2.2.6 Reinforcement Selection

PCI has limited the reinforcement that can be used for the competition. The allowable steel is A615/A706, Welded Wire, Epoxy Coated, A1035, and Plate Steel. Based off of cost and performance, the team has chosen to use A615 because this is all that TPAC has to offer. TPAC offers other reinforcement steel but they do not correlate with the ones allowed with the PCI competition.

2.3 Final Design and Analysis

This was one of the most important tasks that the team completed. Once a cross section was chosen, it had to be perfected. Important elements in this design process were shear design, reinforcement design, loss calculations, the maximum load the beam can bear at its midspan, and the maximum anticipated deflection. Each of these elements allowed the team to make the beam design a little bit better.

2.3.1 Shear Design

The shear design was carried out by hand calculations first and checked in Mathcad. The beam was analyzed for two types of shears which are the flexure shear capacity (Vci) and web shear capacity (Vcw). The hand calculations are shown in appendix K which includes all the equations and methods followed from the ACI code. The ACI codes that were referenced are also mentioned in the hand calculations. In the process of figuring out the shear, the team calculated shears at two points of interest which is at the support that has maximum shear and the load point which has the maximum shear. By comparing these, the maximum shear was selected. The team also checked for the shear component due to steel and it was satisfied. The next step was to figure out the stirrup spacing. Based on the calculations from the shear, the team decided to go with the minimum stirrup requirement and max spacing as per the ACI code. The area required for the shear Av, was satisfied as it reached the expected shear value. The team decided to go with No.3 stirrups at 18" spacing for the beam to handle the factored shear. The calculations ended up getting a safety factor of 2.08 for shear which will be safe for the beam so that it doesn't fail on shear before flexure.

2.3.2 Reinforcement Design

The process of reinforcement design was very much trial, and error based on educated guesses and gaining knowledge from the previous attempt. The team did calculations for designs using both number 3 and number 4 compression steel, and it was determined that number 4 bars best fit the chosen design because the calculations worked out better than number 3 bars. After trial and error, the team calculated that three number 4 bars are the best fit for the beam to crack and break within the given criteria of the competition. This is shown in Appendix F on sheet one. The area of the reinforcing steel and number of strands are different for both the number 3 and 4 bars.

2.3.3 Cracking Load

The cracking load of our beam design was calculated using a MathCAD sheet that the team had previously set up. The MathCAD sheet is located in Appendix F. In the MathCAD sheet the cracking capacity is located on page 6. As per competition guidelines the cracking load of the designed beam must be above 20 kips. Finding a design that met this qualification was difficult at times because whenever a design that met the cracking load criteria, the ultimate capacity would be around 50 or 60 kips, and it could only be between 32 and 40 according to the rules of the PCI Big Beam competition.

The team discovered that adding strands to the bottom of the beam would increase the cracking load, but this would also largely increase the ultimate capacity. Adding compression reinforcement to the design was one of the first alterations that was made in attempt to meet the criteria listed above. Eventually the Big Beam Theory found that for their design, two 0.6-inch strands located at the bottom of the strand and three number four compression steel bars would put the design within desired ranges. This is shown in the shop drawings in Appendix H. The final cracking load the team calculated is 21.083 kips which is within the PCI criteria. Dr. Tuscherer has also informed the team that when steel is added to the beam the cracking load will increase as well.

2.3.4 Max Load at Midspan

The max load at midspan (ultimate capacity) of the beam design was calculated using a MathCAD sheet that the team had previously set up in 476. The MathCAD sheet is located in Appendix F and ultimate capacity is located on page 7. The easiest way the team got the ultimate capacity to be within the given PCI criteria was changing the area of the beam, since the ultimate capacity is changed most by the area of the beam. For the calculations increasing the area, increase the max load at mid span.

As per competition guidelines the max load at mid span of the designed beam must be between 32 and 40 kips. The final ultimate strength the team calculated is 37.062 kips which is within the PCI criteria.

2.3.5 Max Anticipated Deflection

The max anticipated deflection of the beam was calculated in Response 2000. Appendix I shows the beam inputted in response 2000. Response asks for the given properties of the beam and it gives the graphs the stresses of the different components of the beam. Appendix I shows the nine graphs given by response. Those nine graphs are Longitudinal Strain, shrinkage and thermal strain, control moment, longitudinal reinforcement stress, longitudinal reinforcement stress at crack, control moment, longitudinal concrete stress, internal forces, and N and M.

2.4 Predictions

The predictions were calculated in Response 2000. Response 2000 gives the predictions for the cracking and maximum moment at midspan.

2.4.1 Response 2000

Response 2000 is a software that has the functions that help to analyze a reinforced concrete cross-section. Response 2000 can calculate the strength and ductility of the reinforced cross-section (beams) in terms of moment, shear, and axial load. This program works based on the cross-section shape, dimensions, type of reinforcement, and stirrups type and spacing as shown in Appendix I.

2.4.1.1 Cracking and Maximum Moment at Midspan

Response 2000 used to predict the cracking for the chosen cross section and reinforcements (I Beam). The cracking moment calculated using the response 2000 program was determined to be (119.8 kip.ft) as shown in Appendix I. The cracking moment is the moment when the beam starts to crack. Appendix I also shows the maximum moment at midspan using response 2000 and that was predicted to be 144.6 (kip.ft). The maximum moment at midspan is the moment at which the beam breaks. The maximum moment at midspan should be higher than the cracking moment unless the beam cracks and breaks at the same time.

2.4.1.2 Deflection

Deflection predictions are calculated for the prediction of deflection at the ultimate conditions. For the prediction purposes, it is required to use the method of virtual work and find the deflection for the beam with the two ultimate point loads of 18.5 kip each and the distributed self-weight, then integrate it with the moment curvature output from the response. In calculating the deflection by the method of virtual work, following deflection equation is used.

Equation 6. Deflection

$$l(\Delta) = \int_0^l \frac{Mm}{EI} dx$$

Δ = Deflection (in)
M = Internal Moments in the beam in the real diagram (kip*in)
m = Internal moments in the beam in the virtual diagram (kip*in)
E = Modulus of Elasticity (ksi)
I = Effective moment of Inertia (in^4)

To obtain the real moment (**M**) values which occur due to self-weight and live load acting on the beam, section cuts are made for each section of the beam where the loading criteria changes and internal moments are calculated. Similarly, internal moments (**m**) are calculated in the virtual beam with a unit point load. The hand calculations used to obtain these internal moments are given in the Appendix R.

The Response 2000 software will tabulate a moment curvature table for the beam based on its dimensions, losses and strain values. The graph and the raw tabulated data from the Response 2000 is shown in the Appendix P. The curvature value in the tabulated data is equal to **MEI**. This value is directly used in the deflection equation introduced above. **m** values are obtained from the virtual work. **M** values are used to calculate their corresponding curvature values through interpolation using the data obtained from the moment and curvature values from Response 2000. These **M** values are also useful in finding the places of maximum moments which leads to max deflection. The higher the moment, higher the deflection. This effect is shown in the Moment graph in Appendix T. Finally, the integrated values between 4-inch intervals were summed up to find the total deflection of the beam at ultimate. The excel table used for this process of calculation is shown in the Appendix T, which resulted in a total downward deflection of 2.3 inches without the camber. Interpolated curvature values are given in Appendix T.

2.4.1.3 Camber

Camber calculations are important for the deflection predictions because it affects the total deflection of the beam. The camber occurs during the prestressing of the beam with eccentricity. The strands are cut, and the prestress is transferred to the beam. Due to this, a net positive (upward) deflection is created. The net positive camber is obtained using the following equation.

Equation 7. Camber

$$\Delta c = \frac{1}{8} * \frac{PeL^2}{EIe}$$

Δc = Deflection due to Camber (in)
P = Prestressing force (kips)
e = Eccentricity (in)
L = Length of the beam (in)
E = Modulus of Elasticity (ksi)
Ie = Effective moment of Inertia (in^4)

The upward deflection of the beam due to transfer of prestress was calculated to be 0.042 inches. The hand calculations are shown in Appendix R.

2.4.1.4 Total Prestress Losses

Estimating the total prestress losses (TL) can be done by calculating four types of losses which are losses due to elastic shortening (ES), creep of concrete (CR), shrinkage of concrete (SH), and relaxation of tendons (RE); then, sum up all the calculated losses. These losses are needed to estimate the stress and strain on the beam after cutting the tensioned strands (at release stage). After calculating the prestress losses, the stress at release can be calculated by subtracting the total prestress losses from the initial pull stress of the strand (202.6 ksi); thus, the strain can be determined by using the modulus of elasticity equation which is shown below.

Equation 8. Modulus of Elasticity

$$Modulus\ of\ Elasticity\ (E) = \frac{Stress}{Strain}$$

The total losses were calculated to be 22.22 ksi as shown in Appendix P, and the modulus of elasticity of the used strands (grade 270-low relaxation) is 28500 ksi. The strain at release was determined to be 0.00633 as shown in Appendix Q. The strain obtained at release will be used in Response 2000 software to calculate the most accurate moment-curvature diagram which will lead to calculate the most accurate deflection. The total prestress losses were determined using the following equation.

Equation 9. Total Prestress Losses

$$TL = ES + CR + SH + RE$$

2.4.1.4.1 Elastic Shortening

The elastic shortening losses is considered as an immediate loss that occurs on the concrete member when the prestress is just transferred to concrete, the length of the beam will be affected by getting shorter. This loss could also affect the stress of the beam. This stress can be calculated using the following equation; where, ES is the losses due to elastic shortening, Kes is the pretensioned components which is standard as 1.0, Eps is the modulus of elasticity of prestressing tendons, Eci is the modulus of elasticity of concrete at time prestress is applied, and fcir is the net compressive stress in concrete at center of gravity of prestressing force immediately after the prestress has been applied to the concrete. The losses of stress due to elastic shortening was calculated to be 3379.32 psi as shown in Appendix P.

Equation 10. Elastic Shortening

$$ES = \frac{Ees * Eps * fcir}{Eci}$$

2.4.1.4.2 Creep of Concrete

Creep of concrete losses is considered as long-term losses that happen when long-term forces are applied to the beam. In the prestressed beams, the force due the prestress is the long-term force on the beam that could cause creep in the concrete. The creep of concrete was calculated to be 4275.26 psi as shown in Appendix P. This type of losses was determined using the equation below; where, Kcr =1.6 sand-lightweight concrete, fcds is the stress in concrete at center of gravity of prestressing force due to all superimposed, permanent dead loads that are applied to the member after it has been prestressed, Ec is the modulus of elasticity of concrete at 28 days, and the other variables were defined in the elastic shortening section.

Equation 11. Elastic Shortening

$$CR = Kcr * \frac{Eps}{Ec} * (fcir - fcds)$$

2.4.1.4.3 Shrinkage of Concrete

Shrinkage of concrete losses is considered as long-term losses that happen because of drying the concrete which affects the strand; thus, affects the stretch of the strands. In the shrinkage losses, the average annual ambient relative humidity percentage (RH) is used as a function when calculating this loss. The average annual ambient relative humidity percentage can be found based on the area of the construction. The Shrinkage of concrete loss was estimated to be 10286.83 psi as shown in Appendix M. This was calculated using the following equation; where, Ksh = 1.0 for pretensioned components, V/S is the volume-to-surface ratio, RH is the average ambient relative humidity, and Eps is the modulus of elasticity of prestressing tendons.

Equation 12. Shrinkage of Concrete

$$SH = (8.2 * 10^{-6})Ksh * Eps\left(1 - \frac{.06V}{S}\right) * (100 - RH)$$

2.4.1.4.4 Relaxation of Tendons

Relaxation of Tendons losses is considered as long-term losses. This loss occurs in the beam in a long period of time which affects the stress of the tendons by the time. The stress of tendons will reduce by time and cause the beam to lose some stress of the strand due to the relaxation over time. The relaxation of tendons was predicted to be 4282.34 psi as shown in Appendix P and was calculated using the equation below; where, the values for Kre, J, and the coefficient C were taken from tables 5.7.1 and 5.7.2 provided in the PCI design handbook, and the other variables were defined in the previous sections.

Equation 13. Relaxation of Tendons

$$RE = [Kre - J(SH + CR + ES)]C$$

2.5 Shop Drawings

The shop drawings were completed in AutoCAD. The AutoCAD file includes the plan view, dimensions, cross section, and spacing of stirrups. The shop drawings also include the reinforcement details.

2.5.1 AutoCAD

AutoCAD is the program the team used to design the beam. The model view is where the beam was designed. The layout view is where the template is shown. The shop drawings are located in Appendix H

2.5.1.1 Plan View

The plan view shows the skeleton of the beam with its longitudinal dimensions, spacings in between meshes and spacings between the bottom and top concrete and steel. This is located in Appendix H. The longitudinal dimensions of the beam are 22 feet long. The spacing of the stirrups depends on the loading and is shown in the figure located in Appendix H.

2.5.1.2 Dimensions

The Big Beam Theory's final design is a 22-foot-long beam that will be simply supported one foot from each end. An I shaped cross section was selected for the competition with a total height of 24 inches, a maximum width of 23 inches, and a web width of 5 inches. The height of the cross section altered both the calculated cracking load and ultimate load of the beam, so this was taken under serious consideration. Initially the team had desired a beam around 20 inches deep, but the 24 inches allowed the design to better meet the criteria of the competition. Appendix H shows the dimensions of the beam that were designed in AutoCAD.

2.5.1.3 Cross Section

The cross section shows a section of the plan view. The cross section is located in Appendix H The team has 2 different cross sections of the beam. Dr. Tuscherer advised the team to do this, so the dimensions are not overloaded in one drawing. The first cross section (left cross section) does not include the bars or strands added for extra strength. It just shows the area of the I-beam with the dimensions. The second cross section (right cross section) has the bars and strands. It has the locations of the bars and strands with the dimensions.

2.5.1.4 Spacing of Stirrups

The spacing of the stirrups are shown in the plan view. The spacing of the stirrups are 18 inches throughout the beam. At the ends of the beam the stirrups are located 4 inches from the ends. The spacing of the stirrups can be seen in Appendix H.

2.5.2 Reinforcement Details

The reinforcement details are labeled in the shop drawings. As seen in Appendix H, the details are located on the drawings and in the bill of materials. The team is using three number four bars on the top of the beam and two .6-inch strands at the bottom.

3.0 Summary of Engineering Work

This section includes a summary of the engineering work the Big Beam Theory has completed.

3.1 Preliminary Beam Design

The first stage of designing the beam that the team did was preliminary beam design. The initial beam design was started with the creation of a MathCAD sheet. Three MathCAD sheets were created for the T-Beam, I-Beams, and box beam. These sheets calculated values for cracking capacity, ultimate capacity, beam stress, transformed sections at 3 and 28 days, and more. After that a decision matrix was made to select the best beam. The decision matrix included the mix selection, beam type, cost, weight, deflection, and reinforcement selection.

3.2 Final Design and Analysis

The final design and analysis began when the cross section was chosen. After choosing the cross-section, shear design was calculated. The beam was analyzed for two types of shears which are the flexure shear capacity (Vci) and web shear capacity (Vcw). The two places shear was calculated was at a support and load point. The team went with the minimum stirrup requirement that the ACI requires. Then reinforcement was designed for. After trial and error, the team concluded that two 0.6-inch strands located at the bottom of the strand and three number four compression steel bars would put the design within desired ranges.

3.3 Predictions

The predictions were done for deflection at ultimate, camber due to prestressing and losses. These calculations were done in order to get a clear idea of the process of deflection and losses of a prestressed loaded beam and data we are going to analyze after breaking the beam. The total deflection for the beam was obtained by calculating the difference between the camber (upward deflection) and the deflection due to dead and live loads (downward deflection). The total deflection of the beam at ultimate including the camber was calculated to be 2.23 inches which is shown in the excel calculation table in Appendix T.

The losses were calculated and predicted for elastic shortening, creep of concrete, shrinkage of concrete, and relaxation of tendons. The total losses were calculated by summing the four types of losses which was calculated to be 22.22 ksi. This total stress loss was deducted from the initial prestress value to obtain the new strain value to ensure having the most accurate prediction of deflection.

In the process for calculating the deflection the beam was analyzed in depth for its specific loading criteria including the self-weight of the beam. Section cuts were taken at 3 places where the loading criteria changes along the beam and the equations were obtained in terms of x for each section of the beam. These equations obtained from section cuts could be seen in the Appendices R. The x values (length in inches) were substituted for these equations later in the excel sheet in order to obtain the moment values.

The next step was to get the deflection using the equation 6. Here, the equation cannot be used directly for a long span length since the moment curvature curve from the Response 2000 is not a linear curve. Therefore, this calculation was done in MS Excel for every 4 inches of the beam for 240 inches (20ft). This was a tedious process where you need to interpolate for each moment, we have obtained from the equations from virtual work method to moments obtained for the table from Response 2000 in Appendix S. This way we were able to get the accurate moment curvature (M/EI) values to be used in the deflection equation. The 'm' value doesn't change throughout the beam since there is only one-unit virtual load.

Finally, it could be seen in the final column of the table in Appendix T how the integration process was done to obtain the total deflection using the deflection equation for every 4 inch of the beam using the values obtained for M/EI and m.

The camber was calculated directly from the equation obtained from PCI Design Handbook. The values for this equation were obtained directly from the MathCAD sheet.

The predictions will change very slightly due to the continued concrete strengthen past the 28-day cure time. There will be a slight change in the compressive stress (f'c) that will result in a slight change in the prediction. This slight change is negligible as the change is not a major factor. Compressive strength and tensile strength are other factors that have been taken into consideration the day the test will be conducted. These have already been accounted for in the original prediction of 2.23 inches.

3.4 Shop Drawings

The shop drawings were completed in AutoCAD. The AutoCAD file includes the plan view, dimensions, cross section, and spacing of stirrups. The shop drawings also include the reinforcement details. The shop drawing is split into three different drawings; they are a cross section with dimensions, cross section with reinforcement, and an elevation view. The title block includes a bill of material, concrete data, production data, and more details.

3.5 Casting of Beam

TPAC has done the casting of the beam. The beam was poured March 23rd at 9am. The team did not attend the pour of the beam because of the corona virus. It takes the beam 28 days to cure and be ready to be shipped to NAU. The status as of now is that the beam will not be shipped to NAU because the labs are closed and there is no place to store the beam.

3.6 Testing of Beam

The testing of the beam will begin once the labs are opened. The labs are planned to be opened May 1st.

4.0 Summary of Engineering Cost

This section includes a summary of the engineering cost the Big Beam Theory has completed. This includes roles, hours of work, cost, and schedule.

4.1 Roles

It was determined that there were five necessary roles to complete this project: Senior Engineer, Project Engineer, Lab Technician, Engineering Intern, and Administrative Assistant. The Senior Engineer's role consisted of reviewing all documents and processes and competition rules, making the final decision on the beam cross section and concrete mix design, and overseeing all testing. The Project engineer's role consists of overseeing testing and project management and making sure all competition rules are followed. The lab technician is in charge of all laboratory work such as mix design, testing, and ensuring the safety of all in the lab at all times. The Engineering Intern is responsible for helping the Project engineer and Lab Technician when needed. The role of the administrative assistant is to make sure that all documents look professional, that all meetings are scheduled properly, and recording the minutes of all meetings that take place. A majority of the project hours were worked by the engineering intern, followed by the project engineer, lab technician, and the administrative assistant worked the least number of hours.

4.2 Hours of Work

As seen in Appendix M, a table of the breakdown of work completed by each team member for each task for the predicted and actual values. The project engineer and engineering intern spent the most hours on this project, working 197 and 215 hours respectively. The senior engineer had the next most hours as they would be checking in throughout the project to make sure that everything was running smoothly. The lab technician and administrative assistant have the least number of hours as much of the work for this beam design was design based, not lab based. The predicted values and actual values are similar. The corona virus has affected the team's participation for casting the beam as the team did not attend the pour. As of now the team has not tested the beam because the labs are closed, and the beam has yet to arrive to NAU. Once the beam is broke the hours will be even more similar.

4.3 Cost

As seen in Appendix N, a table of costs are broken down for the project. The material costs for the designed beam come out to $432.62, and the cost of labor for the aforementioned 530 hours of labor comes out to $52,670. The cost of labor is estimated to be $675. The total cost of this project comes out to $53,777.62 for all labor and materials listed in the tables below. The predicted cost of project hours prediction and actual are similar. Once the beam is tested the cost will be less similar to the prediction.

4.4 Schedule

Appendix O shows a figure of the updated schedule. It may be noted that this does not match the original schedule, as the team fell behind having not built enough float into the beginning of the semester. On top of getting behind the COVID-19 outbreak and shelter in place recommendation came about and labs on campus got shut down. The team hopes to be able to test the beam when the shelter in place is lifted, as early as the first of May. Should this testing occur the team will still be able to meet the deadline for the PCI Big Beam Competition of July 15, 2020.

5.0 Conclusion

This project was an enlightening experience for all team members, as there is no course offered at Northern Arizona University that covers prestressed concrete. The team is very grateful for their technical advisor Dr. Robin Tuscherer who was there to answer questions and explain aspects of the PCI design handbook as the need arose. Getting to carry out a unique challenge as a team was a learning experience for all members from navigating design handbooks to learning how to use programs such as Response 2000 and MathCAD.

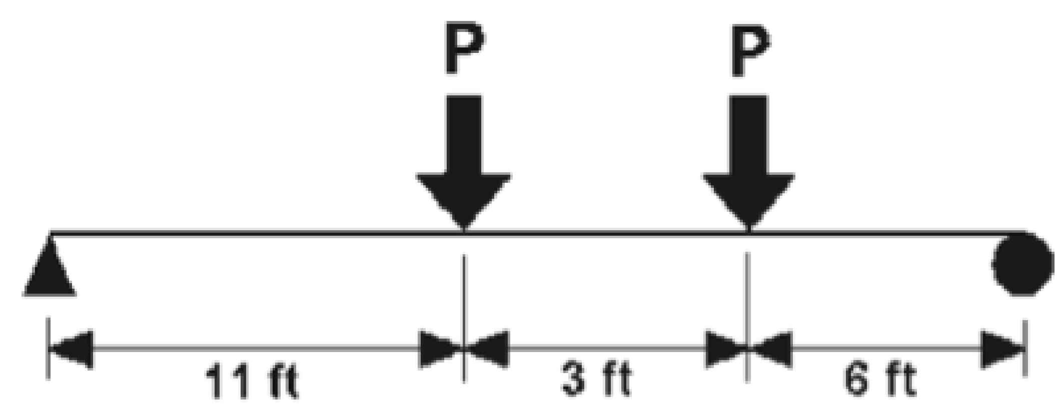
The team is sad to have not been able to test the designed beam due to Covid-19 shelter in place restrictions causing laboratories on campus to be shut down, but remains hopeful that testing will be a possibility come May first if the restrictions are lifted at the end of April as planned.

6.0 References

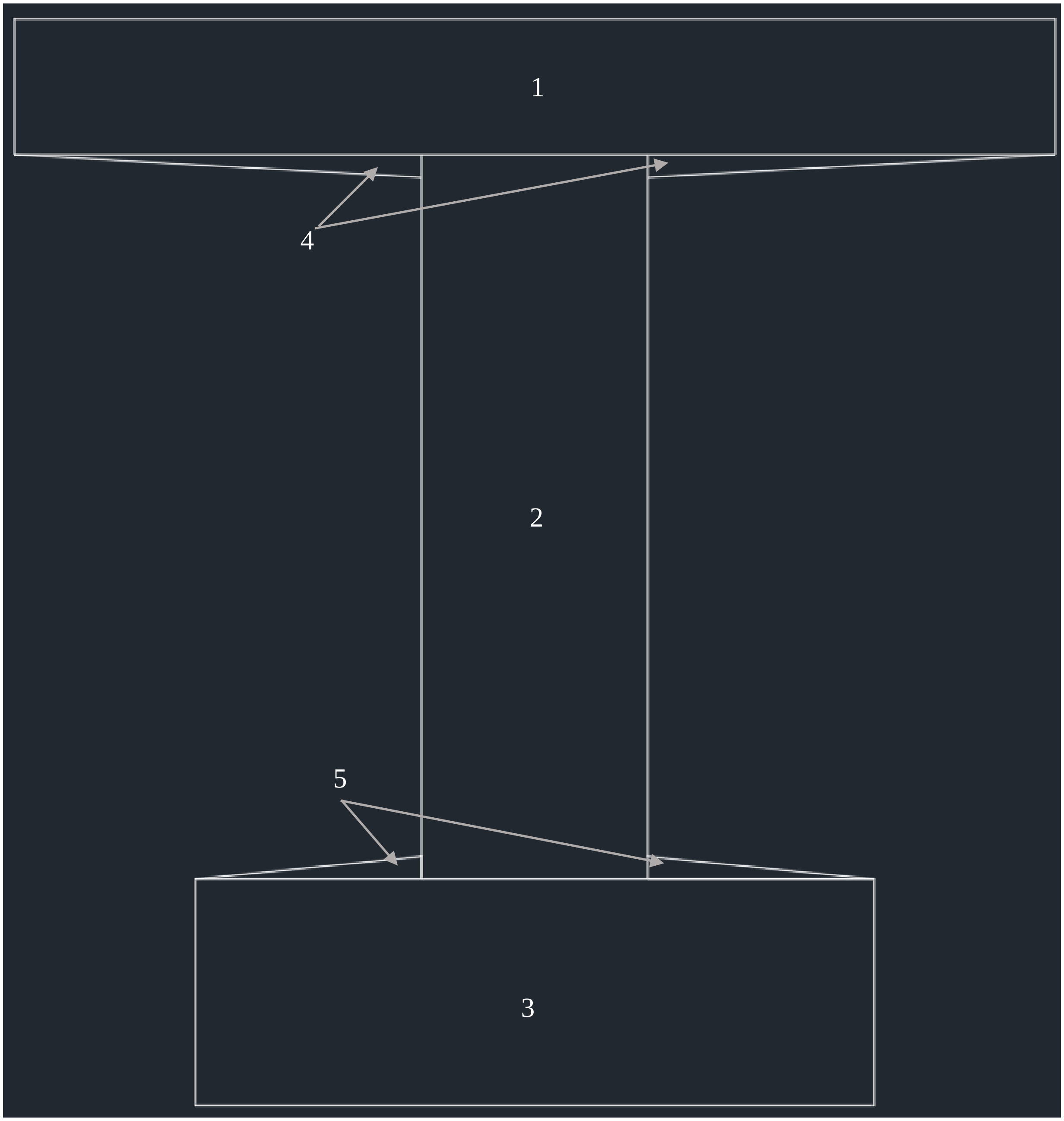
[1] R. Tuchscherer, *Lecture Slides,* Flagstaff: NAU, 2019.
[2] "2019-2020 PCI Competition".
[3] ACI 318-19 Code

7.0 Appendix

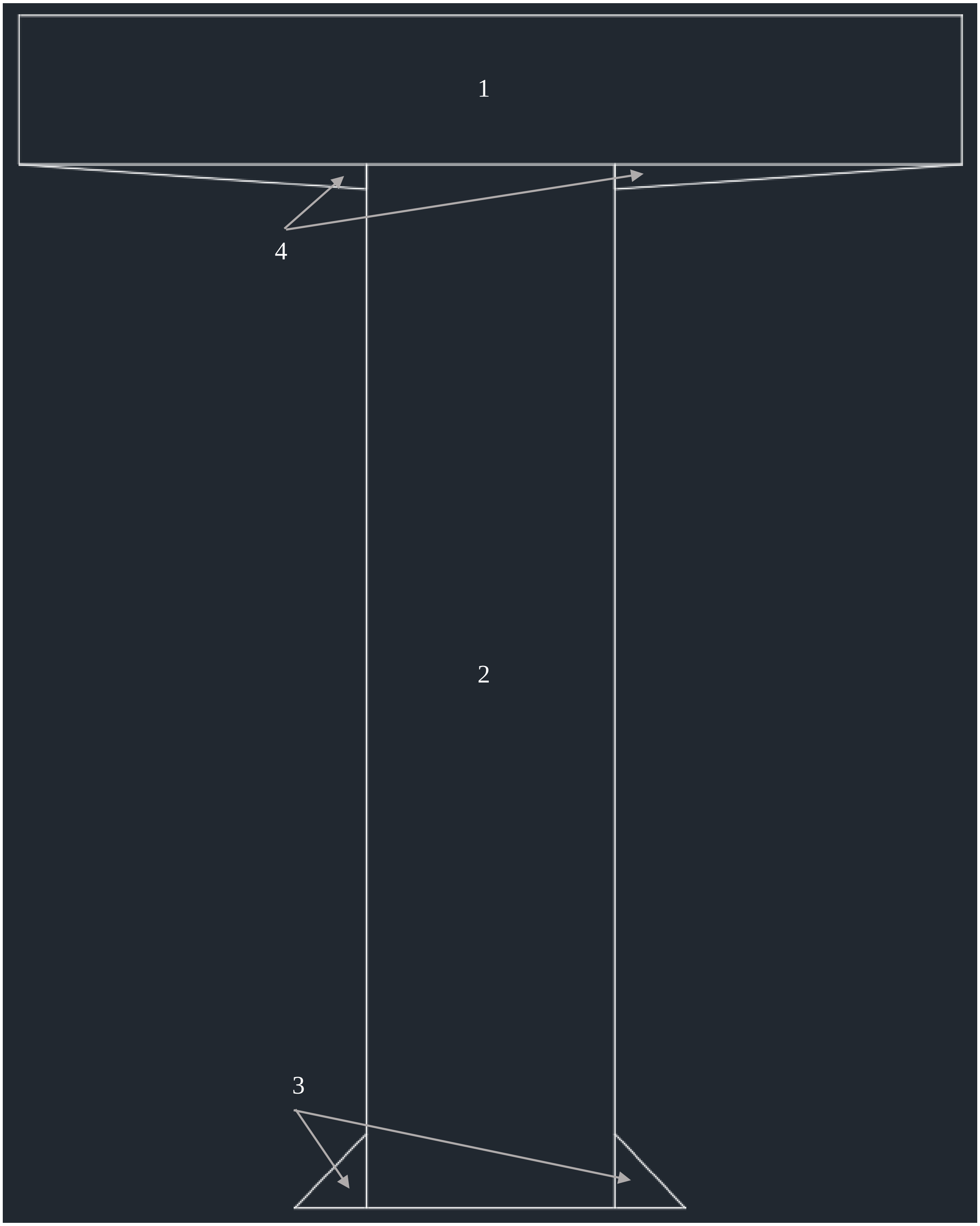
Appendix A – Figure of Applied Loads of Beam



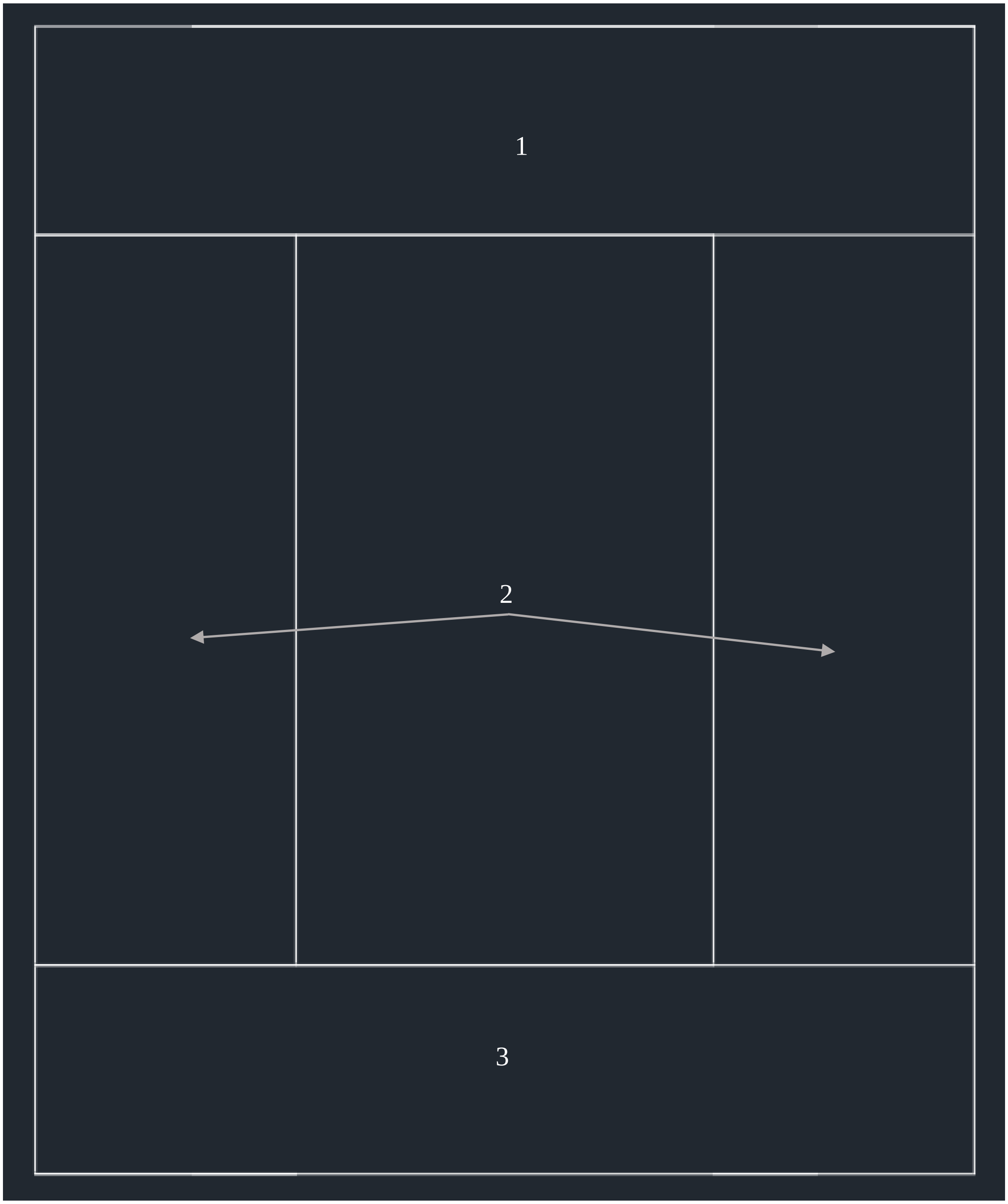
Appendix B – Figure of an I-Beam with Different Areas



Appendix C – Figure of a T-Beam with Different Areas



Appendix D – Figure of a Box Beam with Different Areas



Appendix E – Tables of the Price Breakdown

Concrete	Cost	Notes
Concrete Cost (yd^3)	$\$100 \leq \$20 + \$10$ (concrete strength ksi) $\leq \$200$	Round concrete strength down to nearest ksi
Ultra-High-Performance Concrete	$400/$yd^3$	

Prestressing Strands	Cost	Notes
3/8 Inch Diameter	$0.17/ft	Use estimated lengths used in the beam
1/2 Inch Diameter	$0.30/ft	
1/2 Inch Special	$0.32/ft	
.6 Inch Diameter	$0.42/ft	
.7 Inch Diameter	$0.55/ft	

Steel	Cost	Notes
A615/A706	$0.45/lb	Use estimated lengths and nominal unit weights in this calculation as provided in the PCI Design Handbook
Welded Wire	$0.50/lb	
Epoxy Coated	$0.50/lb	
A1035	$0.70/lb	
Plate Steel	$0.55/lb	

Forming	Cost	Notes
Any Type	$1.25/$ft^2$	Price includes all contact surfaces

Appendix F – Figure of MathCAD Sheets

Given Properties:

1

Area of Reinforcing Steel	$Asprime := 3 \cdot 0.2in^2 = 0.6 \cdot in^2$
Tensile Strength of Steel Reinforcement	$fy := 60ksi$
No. of Strands	$N := 3$
Area of Strand	$Ap := 2 \cdot 0.153in^2 = 0.306 \cdot in^2$
Tensile Stress of Strand at Release	$fpi := 174ksi$
Tensile Stress of Strand at Cracking	$fcr := 180ksi$
Tensile Stress of Strand at Ultimate	$fu := 265$
Compressive Strength of Concrete at 3 days	$fc_3 := 5ksi$
Compressive Strength of Concrete at 28 days	$fc_{28} := 8ksi$
Modulus of Elasticity at 3 days	$Ec_3 := 57ksi \cdot \sqrt{\frac{fc_3}{psi}} = 4.031 \times 10^3 \cdot ksi$
Modulus of Elasticity at 28 days	$Ec_{28} := 57ksi \cdot \sqrt{\frac{fc_{28}}{psi}} = 5.098 \times 10^3 \cdot ksi$
Modulus of Elasticity of Steel	$Es := 29000ksi$
Modulus of Elasticty of Strand	$Ep := 28500ksi$
Unit Weight of Reinforced Concrete	$\gamma c := 121 \frac{lbf}{ft^3}$
Unit Weight of Steel	$\gamma s := 490 \frac{lbf}{ft^3}$

Section Properties:

$i := 1..7$

Width: Height: Area: Moment of Inertia:

$b_1 := 23in \quad h_1 := 3in \quad A_1 := b_1 \cdot h_1 = 69 \cdot in^2 \quad I_1 := b_1 \cdot \frac{(h_1)^3}{12} = 51.75 \cdot in^4$

$b_2 := 5in \quad h_2 := 16in \quad A_2 := b_2 \cdot h_2 = 80 \cdot in^2 \quad I_2 := b_2 \cdot \frac{(h_2)^3}{12} = 1.707 \times 10^3 \cdot in^4$

$b_3 := 15in \quad h_3 := 5in \quad A_3 := b_3 \cdot h_3 = 75 \cdot in^2 \quad I_3 := b_3 \cdot \frac{(h_3)^3}{12} = 156.25 \cdot in^4$

$b_4 := 9in \quad h_4 := .5in \quad A_4 := \frac{(b_4 \cdot h_4)}{2} = 2.25 \cdot in^2 \quad I_4 := b_4 \cdot \frac{(h_4)^3}{12} = 0.094 \cdot in^4$

$b_5 := 5in \quad h_5 := .5in \quad A_5 := \frac{(b_5 \cdot h_5)}{2} = 1.25 \cdot in^2 \quad I_5 := b_5 \cdot \frac{(h_5)^3}{12} = 0.052 \cdot in^4$

$H := h_1 + h_2 + h_3 = 24 \cdot in$

$$Agconcrete := \sum_{i=1}^{5} A_i = 227.5 \cdot in^2$$

Centroid:

$y_1 := h_3 + h_2 + \left(\frac{h_1}{2}\right) = 22.5 \cdot in$

$y_2 := h_3 + \left(\frac{h_2}{2}\right) = 13 \cdot in$

$y_3 := \frac{h_3}{2} = 2.5 \cdot in$

$y_4 := h_3 + h_2 - \frac{h_4}{2} = 20.75 \cdot in$

$y_5 := h_3 + \frac{h_5}{2} = 5.25 \cdot in$

$y_6 := \frac{h_3}{2} = 2.5 \cdot in$

$y_7 := h_3 + h_2 + \left(\frac{h_1}{2}\right) = 22.5 \cdot in$

$$\sum_{i=1}^{7} y_i = 89 \cdot in$$

Transformed Section at 3 Days:

$$n_{3s} := \frac{Es}{Ec_3} = 7.195$$

$$n_{3p} := \frac{Ep}{Ec_3} = 7.071$$

$$A_6 := (n_{3s} - 1)\cdot Asprime = 3.717\cdot in^2$$

$$A_7 := (n_{3p} - 1)\cdot Ap = 1.858\cdot in^2$$

$$Atr_3 := \sum_{i=1}^{7} A_i = 233.075\cdot in^2$$

$$I_6 := 0$$

$$I_7 := 0$$

$$ybar3 := \frac{\left[\sum_{i=1}^{7} (A_i\cdot y_i)\right]}{\left(\sum_{i=1}^{7} A_i\right)} = 12.375\cdot in \qquad \sum_{i=1}^{7} I_i = 1.915\times 10^3\cdot in^4$$

$$d_1 := ybar3 - y_1 = -10.125\cdot in$$

$$d_2 := ybar3 - y_2 = -0.625\cdot in$$

$$d_3 := ybar3 - y_3 = 9.875\cdot in$$

$$d_4 := ybar3 - y_4 = -8.375\cdot in$$

$$d_5 := ybar3 - y_5 = 7.125\cdot in$$

$$d_6 := ybar3 - y_6 = 9.875\cdot in$$

$$d_7 := ybar3 - y_7 = -10.125\cdot in$$

$$\sum_{i=1}^{7} d_i = -2.374\cdot in$$

$$Itr_3 := \sum_{i=1}^{7} \left[I_i + A_i\cdot (d_i)^2\right] = 1.711\times 10^4\cdot in^4$$

Transformed Section at 28 Days:

$$n_{28s} := \frac{Es}{Ec_{28}} = 5.688$$

$$n_{28p} := \frac{Ep}{Ec_{28}} = 5.59$$

$$A_6 := \left(n_{28s} - 1\right)\cdot Asprime = 2.813\cdot in^2$$

$$A_7 := \left(n_{28p} - 1\right)\cdot Ap = 1.405\cdot in^2$$

$$Atr_{28} := \sum_{i=1}^{7} A_i = 231.718\cdot in^2$$

$$ybar28 := \frac{\left[\sum_{i=1}^{7} \left(A_i \cdot y_i\right)\right]}{\left(\sum_{i=1}^{7} A_i\right)} = 12.394\cdot in \qquad \sum_{i=1}^{7} I_i = 1.915\times 10^3\cdot in^4$$

$$d_1 := ybar28 - y_1 = -10.106\cdot in$$

$$d_2 := ybar28 - y_2 = -0.606\cdot in$$

$$d_3 := ybar28 - y_3 = 9.894\cdot in$$

$$d_4 := ybar28 - y_4 = -8.356\cdot in$$

$$d_5 := ybar28 - y_5 = 7.144\cdot in$$

$$d_6 := ybar28 - y_6 = 9.894\cdot in$$

$$d_7 := ybar28 - y_7 = -10.106\cdot in$$

$$\sum_{i=1}^{7} d_i = -2.243\cdot in$$

$$Itr_{28} := \sum_{i=1}^{7} \left[I_i + A_i\cdot\left(d_i\right)^2\right] = 1.697\times 10^4\cdot in^4$$

Beam Stress at Release:

$fpi := 174ksi \quad Fpi := fpi \cdot Ap = 53.244 \times 10^{0} \cdot kip$

$fcr := 180ksi$

$fu := 265ksi$

Strand Force Eccentricity $\quad e := ybar3 - y_3 = 9.875 \cdot in \quad ybar3 = 12.375 \cdot in$

Axial Stress $\quad \sigma a := \dfrac{-Fpi}{Atr_3} = -228.442 \cdot psi$

Flexural Stress at the Bottom $\quad bot\ \sigma f := \dfrac{[(Fpi \cdot e) \cdot ybar3]}{Itr_3} = 380.347 \cdot psi$

Flexural Stress at the Top $\quad top\ \sigma ft := \dfrac{[(Fpi \cdot e) \cdot (H - ybar3)]}{Itr_3} = 357.285 \cdot psi$

Stress at the Bottom $\quad \sigma a - \sigma f = -0.609 \cdot ksi$

Stress at Top $\quad \sigma a + \sigma ft = 128.844 \cdot psi$

Allowable Compressive Stress $\quad .7 \cdot fc_3 = 3.5 \cdot ksi$

Allowable Tensile Stress $\quad 6 \cdot \left[\sqrt{(fc_3 \cdot psi)}\right] = 0.424 \cdot ksi$

$$Check1 := \begin{vmatrix} \text{"Okay"} & \text{if } .7 \cdot fc_3 \geq |\sigma a - \sigma f| \\ \text{"No Good"} & \text{if } .7 \cdot fc_3 < |\sigma a - \sigma f| \end{vmatrix}$$

Check1 = "Okay"

$$Check2 := \begin{vmatrix} \text{"Okay"} & \text{if } .6\left[\sqrt{(fc_3 \cdot psi)}\right] \geq |\sigma a - \sigma ft| \\ \text{"Provide Crack Control Reinforcement"} & \text{if } .6 \cdot \left[\sqrt{(fc_3 \cdot psi)}\right] < |\sigma a + \sigma ft| \end{vmatrix}$$

Check2 = "Provide Crack Control Reinforcement"

Cracking Capacity:

$$\omega sw := (Agconcrete \cdot \gamma c) = 191.163 \cdot \frac{lbf}{ft}$$

$$L := 20ft$$

$$Msw := \frac{\left(\omega sw \cdot L^2\right)}{8} = 9.558 \cdot ft \cdot kip$$

$$\sigma sw := Msw \cdot \frac{ybar28}{Itr_{28}} = 83.755 \cdot psi$$

$$fcr := 7.5psi \sqrt{\frac{fc_{28}}{psi}} = 670.82 \cdot psi$$

$$MLL := 1kip \cdot in$$

Given

$$fcr = \left[-\sigma a + \sigma sw - \sigma f + \frac{(MLL \cdot ybar28)}{Itr_{28}}\right]$$

$$MLL := Minerr(MLL)$$

$$Pcr := \frac{2 \cdot (MLL)}{8ft} = 21.083 \cdot kip$$

$$MLL = 84.332 \cdot kip \cdot ft$$

Ultimate Capacity

$d := y_1 = 22.5 \cdot in$

$dprime := y_6 = 2.5 \cdot in$

$$\beta := \begin{cases} 0.85 & \text{if } (fc_{28} \le 4000psi) \\ \left[0.85 - \left[0.5 \cdot \left(\dfrac{fc_{28} - 4000psi}{1000psi}\right)\right]\right] & \text{if } 4000psi < fc_{28} < 8000psi \\ 0.65 & \text{if } fc_{28} \ge 8000psi \end{cases} = 0.65$$

Initial Guess: $c := 1in$ Strain in Concrete at Failure: $\varepsilon := .003$

Given

$$\left(0.85 \cdot fc_{28} \cdot \beta \cdot c \cdot b_1\right) + \min\left[Asprime \cdot \varepsilon \cdot \left[\frac{(c - dprime)}{c}\right] \cdot Es, fy \cdot Asprime\right] - Ap \cdot fu = 0$$

$c := \text{Minerr}(c) = 1.284 \cdot in$ $h_3 = 5 \cdot in$

$$\text{Check in Flange} := \begin{cases} \text{"Okay"} & \text{if } c \le h_3 \\ \text{"Incorrect Assumption"} & \text{if } c > h_3 \end{cases}$$

Check in Flange = "Okay"

$Cc := 0.85fc_{28} \, \beta \cdot c \cdot b_1 = 130.528 \cdot kip$ $\varepsilon = 3 \times 10^{-3}$

$$Cs := Asprime \cdot Es \cdot \varepsilon \cdot \left[\frac{(c - y_6)}{c}\right] = -49.438 \cdot kip$$

$T := Ap \cdot fu = 81.09 \cdot kip$

$Mn := fu \cdot Ap \cdot [d - (\beta \cdot c \cdot 0.5)] + Cs \cdot (\beta \cdot c \cdot 0.5 - dprime) = 157.804 \cdot ft \cdot kip$

$$Pn := \frac{[(Mn - Msw) \cdot 2]}{8ft} = 37.062 \cdot kip$$

Appendix G – Table of Calculations for Cost and Weight

	Cost ($)	Unit
Concrete	100	yd^3
Strand	0.42	ft
Compression Steel	0.5	lb
Formwork	1.25	ft^2

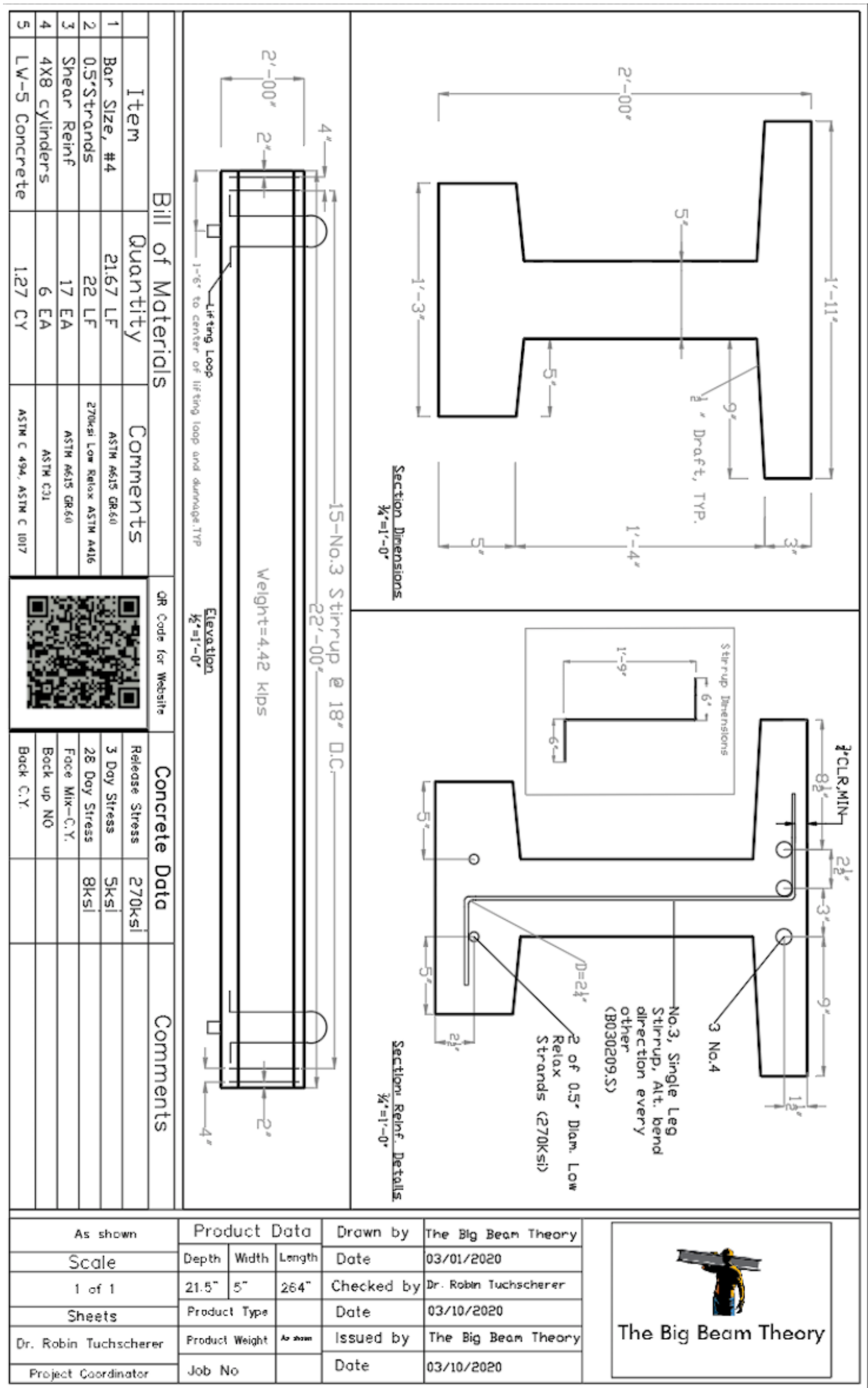
Initial I Beam			Cost ($)	Weight	
Length	20	ft		4722.361111	lbf
Gross Area (Ag)	281	in^2	144.5473251	4.722361111	kip
Steel Area (As)	0.668	lb/ft	6.68		
Prestressing Area (Ap)	0.306	in^2	2.5704		
Perimiter	87.66	in			
Formwork	146.1	ft^2	182.625		
		Total Cost	336.4227251		

Initial T Beam			Cost ($)	Weight	
Length	20	ft		6083.611111	lbf
Gross Area (Ag)	362	in^2	186.2139918	6.083611111	kip
Steel Area (As)	0.668	lb/ft	6.68		
Prestressing Area (Ap)	0.459	in^2	3.8556		
Perimiter	34	in			
Formwork	56.66666667	ft^2	70.83333333		
		Total Cost	267.5829251		

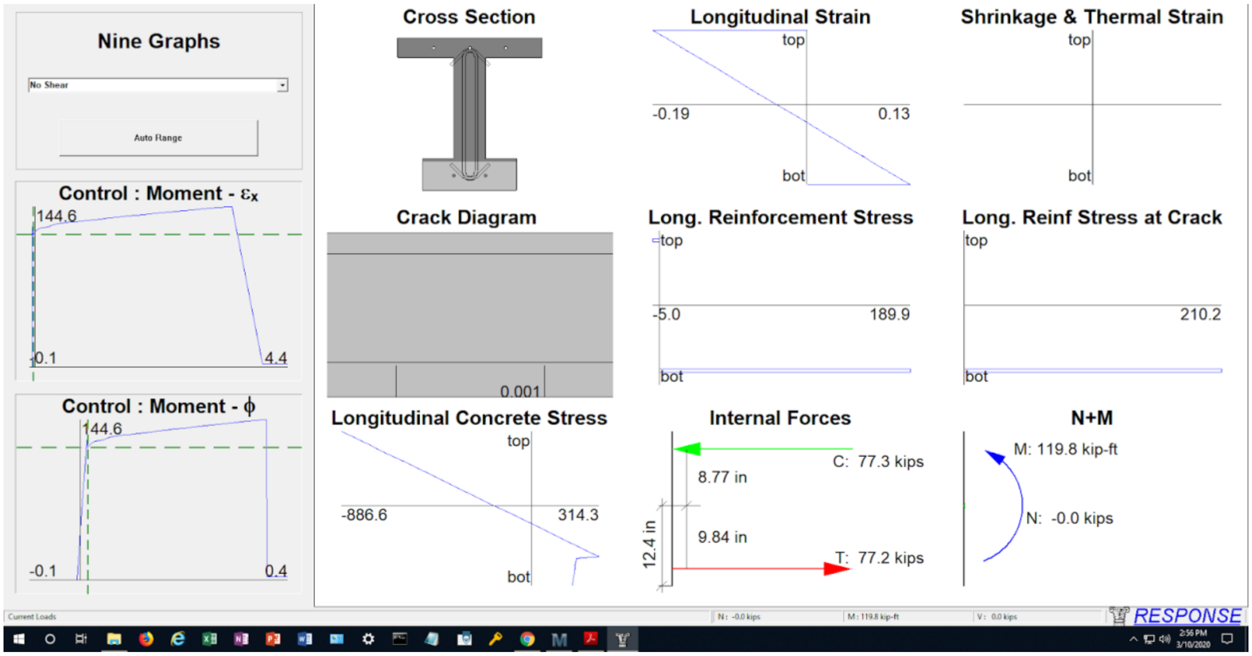
Initial Hollow Box Beam			Cost ($)	Weight	
Length	20	ft		6285.277778	lbf
Gross Area (Ag)	374	in^2	192.3868313	6.285277778	kip
Steel Area (As)	0.668	lb/ft	6.68		
Prestressing Area (Ap)	0.459	in^2	3.8556		
Perimiter	57	in			
Formwork	95	ft^2	118.75		
		Total Cost	321.6724313		

Final I Beam			Cost ($)	Weight	
Length	22	ft		4205.590278	lbf
Gross Area (Ag)	227.5	in^2	187.7304098	4.205590278	kip
Steel Area (As)	0.668	lb/ft	7.348		
Prestressing Area (Ap)	0.306	in^2	2.82744		
Perimiter	87.66	in			
Formwork	160.71	ft^2	200.8875		
		Total Cost	398.7933498		

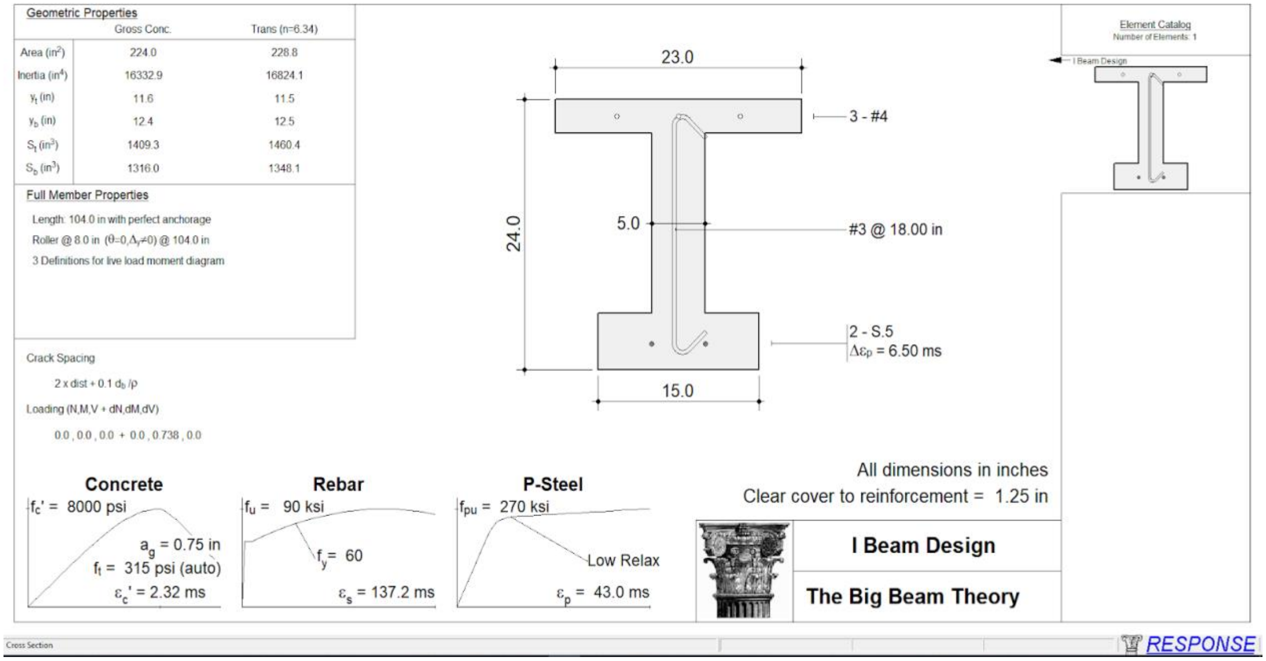
Appendix H – Figure of Shop Drawings



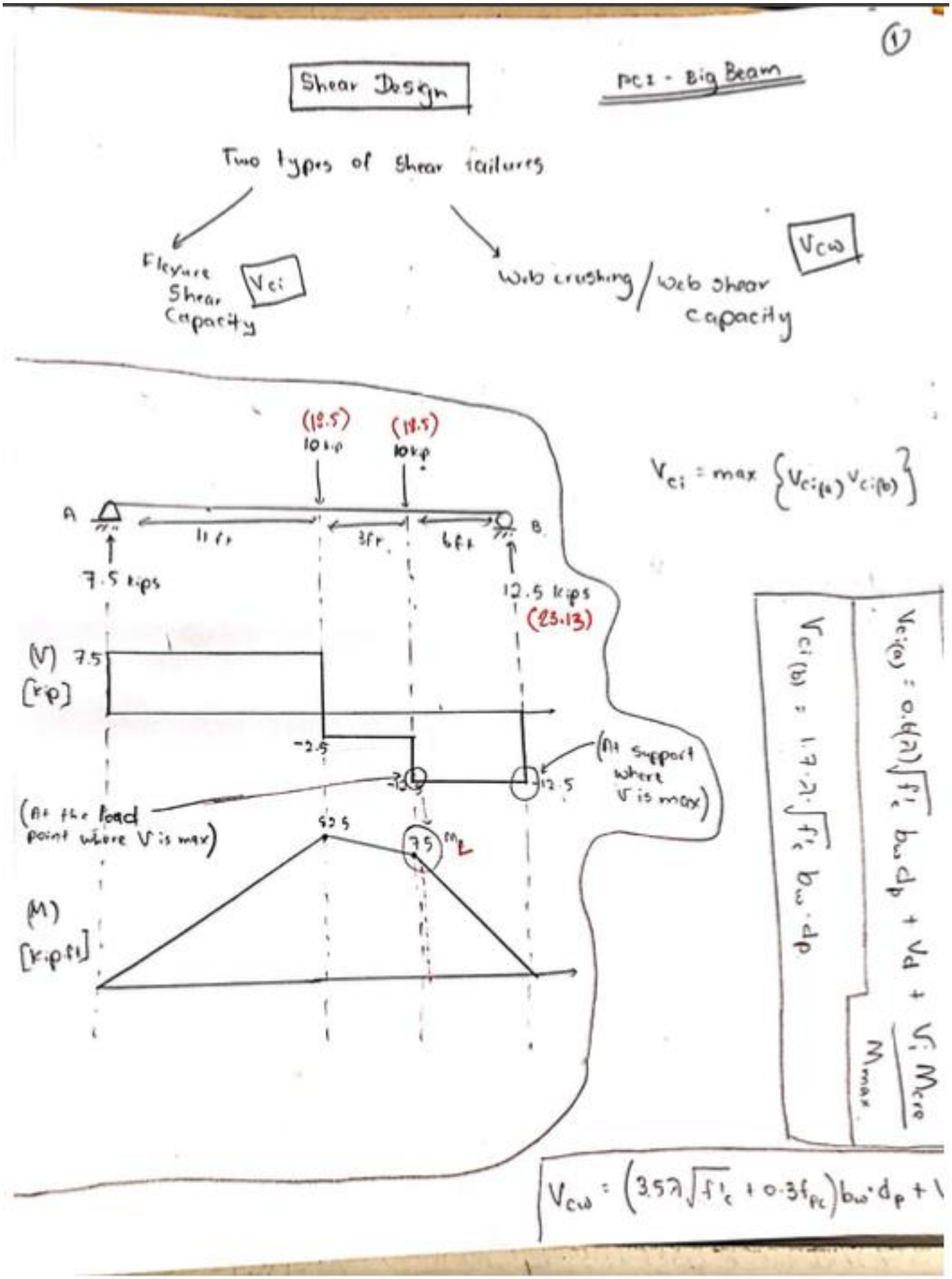
Appendix I – Figure of Response 2000 Graphs



Appendix J – Figure of Response 2000 Cross Section



Appendix K – Figure of Hand Calculations of Shear and Size and Spacing of Stirrups



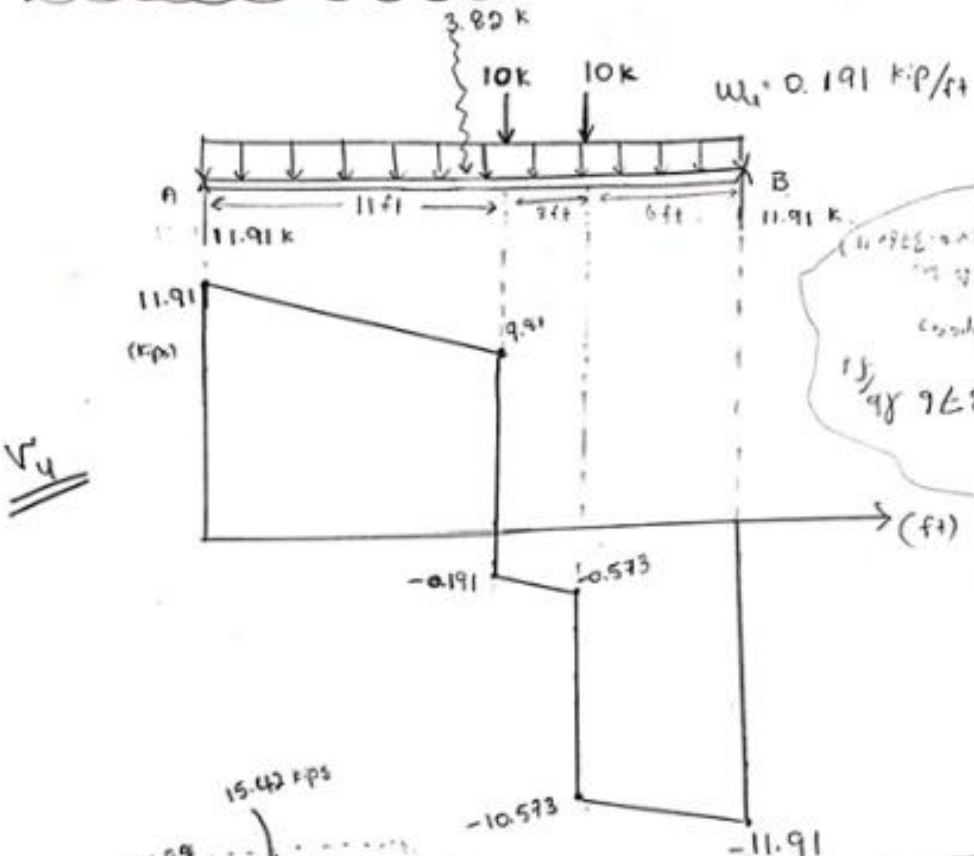
$V_{ci} = \max \{V_{ci(a)}\ V_{ci(b)}\}$

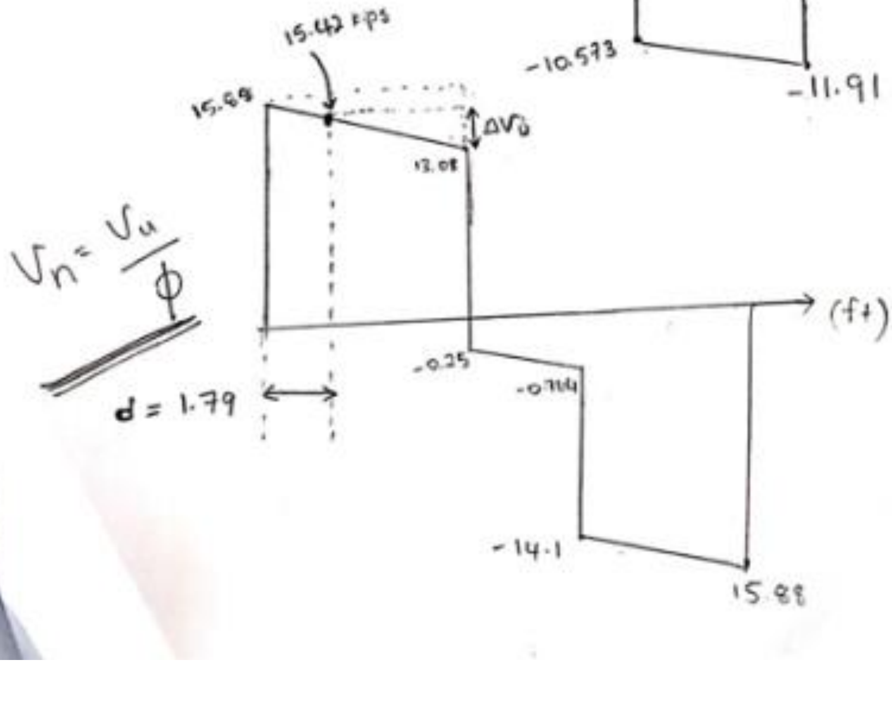
$V_{ci(a)} = 0.6(\lambda)\sqrt{f'_c}\, b_w d_p + V_d + \frac{V_i M_{cre}}{M_{max}}$

$V_{ci(b)} = 1.7 \cdot \lambda \cdot \sqrt{f'_c}\, b_w \cdot d_p$

$V_{cw} = (3.5\lambda\sqrt{f'_c} + 0.3 f_{pc}) b_w \cdot d_p + V$

* V_u diagram with s.w & point loads combined:





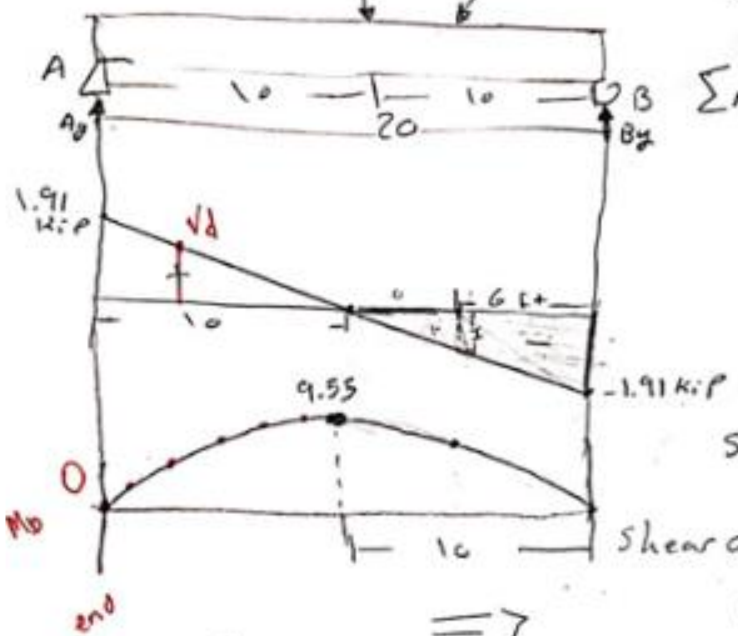
using Sim Δ's

$$\frac{\Delta V_u}{9.21} = \frac{2.8}{11}$$

$$\Delta V_u = 2.34$$

$$\therefore \frac{V_u}{\phi} = 15.42^k$$

3.82 kip $\quad w_{sw} = 191.163 \frac{lb}{ft} = 0.1912 \frac{kip}{ft}$



$\sum M_B = 0; = -A_y(20) + (3.82)(10) = 0$

$\boxed{A_y = 1.91 \text{ kip}}$

$\boxed{\Rightarrow B_y = 1.91 \text{ kip}}$

Sim Δ

$\text{Shear at}(6ft) = \frac{10\ ft}{1.91\ kip} = \frac{6\ ft}{x\ kip}$

$\Rightarrow$

$\boxed{\Rightarrow 1.146 \text{ kip}}$

1146 lb

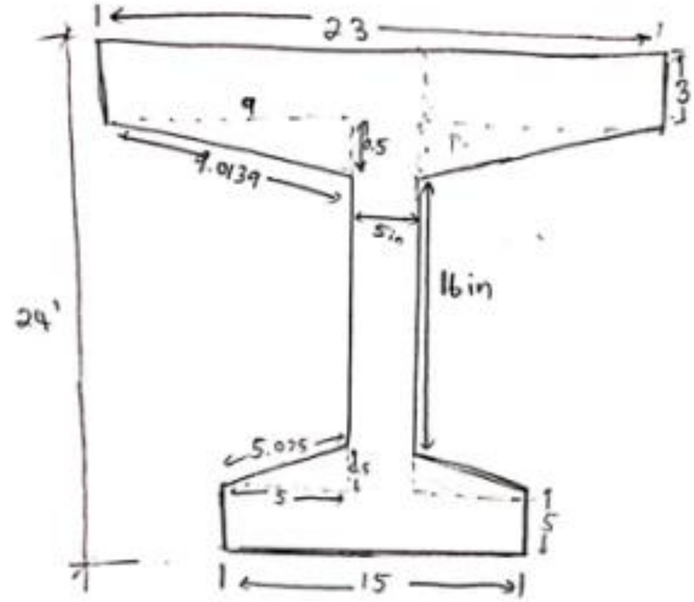
$\text{Moment} = (0.5)(6ft)(1.146)$
$= 3.438 \text{ kip.ft}$

$\Rightarrow M(6ft) = 9.55 - 3.438$
$= \underline{6.11 \text{ kip.ft}}$

Moments & Shears due to self weight

$P_{cr} = 21.083$ kip

$P_n = 37.062$ kip



① Calculating of shear near load point where shear is max

* calculate cracking moment at a distance 6ft from support B.

$$M_{cre} = \left[\frac{\left(I_{1,25}\right)}{\bar{y}}\right] \times \left[6\lambda\sqrt{f'_c} + f_{pe} - f_{d(x)}\right]$$

bottom

$$f_{d(6ft)} = \frac{M_d \cdot \bar{y}}{I_g}$$

$$= \frac{(75)\times 12)\times 12.39\ (in)}{1.697\times10^4\ in^4}$$

$$f_d = 0.657\ psi$$

0 at end

$$= \left[\frac{1.697\times10^4\ in^4}{12.394\ in}\right] \times \left[6(1)\sqrt{8000} + \underbrace{0.614}_{\text{Compressive stress at bottom}} + 0.657\right]$$

8000 psi

75 kip ft × 12

≈ 736 K·in

$$M_{cre} = 22015.61\ k\text{-}in$$

* Calculating the concrete flexure-shear capacity at 6ft from support B

(1) * $V_{ci_{(a)}} = 0.6(\lambda)\sqrt{f'_c}\, b_w \cdot d_p + V_d + \dfrac{V_i \cdot M_{cre}}{M_{max}}$ (5.27)

(a): $0.6(\cancel{1})^{0.95}\sqrt{8000}(5)(21.5) + 1146 + \dfrac{(26\,650)(24974390)}{826800}$

$(psi)\ (in)\ (in)$ ... $(lb\text{-}in)$ ... $(lb\text{-}in)$

$V_{ci} = \underline{811.81 \text{ Kips}}$ → conside.

$(\approx 26.5\text{kip}) \times 0.95 = \underline{25K} \lessapprox 2.7\sqrt{f'_c}\, b_w d$ control

P = failure load

$V_i = V_u - V_d$

$V_i = \cancel{0.75P} - V_d$ (23.13)

$V_i = a\,\cancel{25}(\cancel{37.06})$ (23.13)

-1.146

$V_i = \underline{26.65 \text{ kips}}$ (22)

(2) * (b) $V_{ci_{(b)}} = 1.7\lambda\sqrt{f'_c}\, b_w \cdot d_p = 1.7(1)\sqrt{8000} \times (5 \times 21.5)$

$= \cancel{16.345 \text{ kips}}$

(c) $2(0.95)\sqrt{8000}(5'')(21.5'') = 18.3\text{kip}$

$V_{ci(a)} > V_{ci(b)}$?

$A_{ps} \cdot f_{se} = (2)(0.153 in^2)(174 ksi) = 53\text{kip} > 31.8\text{kip}$

$0.4 A_{ps} f_{pu} = 0.4(2)(0.153in^2)(260ksi) = 31.8\text{kip}$

$M_{max} = M_u - M_d$

(from show) $75 - 6.11$

$M_{max} = 68.9$ kpft

$M_{max} = \underline{826.9 \text{ kpin}}$

$d_p = 21.5''$

* Calculate web shear capacity

$V_{cw} = (3.5\lambda\sqrt{f'_c} + 0.3 f_{pc})\, b_w \cdot d_p$

$V_{cw} = \left(3.5(\cancel{1})^{0.95}\sqrt{8000} + 0.3(174)10^3\right)\dfrac{5 \times 21.5}{1000} > V_{ci}$ 53000psi

$V_{cw} = \underline{5645.15 \text{ kips}}$

$f_{pc} \approx \cancel{174}ksi$

$f_{pc} = \dfrac{P}{A} = \dfrac{(174ksi)(2)(0.153in^2)}{232 in^2}$

$= 53$ ksi

* Minimum from V_{ci} & V_{cw} is selected as concrete shear.

$$\therefore V_c = \min\{V_{ci}, V_{cw}\}$$

$$V_{ci} = 811.8 \text{ kips}$$

Shear component due to steel

if #3@18" works, then Av = (0.11in²)

$$V_s = \frac{A_v \cdot f_y \cdot d}{s} = \frac{(0.6)(60)(21.5)}{5} = 154.8 \text{ kips}$$

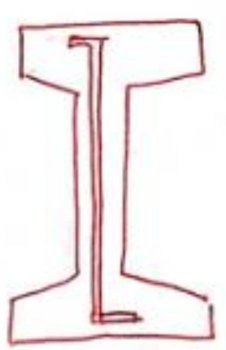
* Total shear Resistance at 6ft from right $= V_n = V_c + V_s$

$$= 811.8 \text{ kips} + 154.8 \text{ kips}$$

$$= 966.6 \text{ kips}$$

$$\phi V_n = \phi(V_c + V_s) = 0.75 \times 966.6 = 724.95 \text{ kips}$$

Spa < $3h/4$ = 18" (Spacing max)



(5) Calculating of Shear load point where shear is max at the support B. (6)

* Cracking moment at Support B

$$M_{cre} = \left[\frac{I_{t,28}}{\bar{y}}\right] \times \left[6\lambda\sqrt{f'_c} + f_{pe} - \cancel{f_{d(t)}}^{\,0}\right]$$

$$= \left[\frac{1.697\times10^{4}}{12.394}\right] \times \left[6(1)\sqrt{8} + 0.614\right]$$

$$M_{cre} = \underline{\underline{24076.98}}\ \text{k}\cdot\text{in}$$

* concrete flexure shear Capacity at support

$$(1)\quad V_{ci} = 0.6(\lambda)\sqrt{f'_c}\cdot b_w\cdot d_p + V_d + \frac{V_i\,M_{cre}}{M_{max}}$$

$$V_{c_i} = 0.6\,(1)\sqrt{8000}\,(5)(21.5) + \cancel{1910}^{\,0} + \cancel{\frac{25890\times24076980}{0}}^{\,0}$$

$$V_{ci} = \underline{\underline{7.679\ \text{kips}}}$$

$V_i = V_u - V_d$

$V_i = 0.95(37.06)$

-1.91

$V_i = \underline{\underline{25.89\ \text{k}}}$

$M_{max} = M_u - M_d$

$= 0 - 0$

$= 0$

$$(2)\quad V_{ci} = 1.7\cdot\lambda\cdot\sqrt{f'_c}\cdot b_w\cdot d_p = 1.7(1)\sqrt{8000}\times(5\times21.5)$$

$$= \underline{\underline{16.345\ \text{kips}}} \longrightarrow \text{Consider.}$$

* Calculate web shear capacity

$$V_{cw} = \left(3.5\,\lambda\sqrt{f'_c} + 0.3 f_p\right) b_w \cdot d_p.$$

$$V_{cw} = \left(3.5(1)\sqrt{5000} + 0.3(174\times10^3)\right) \times \frac{5\times 21.5}{1000}$$

$$V_{cw} = \underline{\underline{5645.15 \text{ kips}}}$$

Min $\longrightarrow$ $V_{ci} \longrightarrow$ 16.345 kips ✓

Shear due to steel at support

$$V_s = \frac{A_v \cdot f_y \cdot d}{s} = \frac{0.6(60)\,21.5}{5} = 154.8 \text{ kips}$$

Total shear resistance at B $= V_n = V_c + V_s$

$$= 16.345 + 154.8$$

$$V_n = \underline{\underline{171.15 \text{ kips}}}$$

$$\phi V_n = \phi\left(V_c + V_s\right) = 0.75 \times 171.15 = \underline{\underline{128.36 \text{ kips}}}$$

Stirrup Spacing

— Big Beam Theory

$h/2 \rightarrow$ for prestressed concrete.

Locations of interest

(i) Max V_u

(ii) $V_u/\phi \leq V_c$ (min stirrups / max spacing)

(iii) $V_u/\phi \leq V_c/2$ (no stirrups)

*Concrete contribution to shear strength : $V_c = 811.81$ kips

* Determine the shear capacity of stirrups required.

$$V_{s,req} \geq \frac{V_u}{\phi} - V_c$$

$$\geq 15.42 - 25.11$$

if we use calculated V_c it's gonna give a (-)ve value

min. Stirrups

- $A_{v,min}$/ spa. $\geq$ Table 9.6.3.4
- max. spa. $\leq$ 3h/4 check 9.7.6.2.2

* Minimum from V_{ci} & V_{cw}. is selected

$$\therefore V_c = \min\{V_{ci}, V_{cw}\}$$

$V_{ci} = 25.18$ kips

Stirrups req

There are no stirrups required if your $\frac{V_u}{\phi} \leq V_c$

ACI Table 9.6.3.4

[min stirrups/ max. spacing]

check

$$\frac{Av_{min}}{Spa} \geq$$

Max spacing $\leq$

* Start with like
→ 1 #3 at 18 in
checks that meet $\frac{Av_{min}}{space}$ ✓ if
does. Then that's all you need.

* Spacing has to be less than $\frac{3H}{4} = 18''$

[use 1 #3 for minimum] ← * If see if #3 at 18"
If it satisfies, then A_v small

Shear Calcs – Final

(1) * At near load point where shear is max;

(1) $M_{cre} = \left[\frac{I_{t,x}}{\bar{y}}\right] \times \left[6\lambda\sqrt{f'_c} + \cancel{f_{pe}}^{\,396\,psi} - \cancel{f_{d(x)}}^{\,negligible}\right]$ $\boxed{ACI\ 22.5.6.3.1d}$

$= \left[\frac{1.697 \times 10^4}{12.394}\right] \times 6(0.95)\sqrt{8000} \approx \underline{698.05\ kip \cdot in}$

(2) $\boxed{ACI\ 22.5.6.3.1}$

(a) $V_{ci} = (0.6)\lambda\sqrt{f'_c} \cdot b_w \cdot d_p + V_d + \frac{V_i M_{cre}}{M_{max}}$ (ACI 22.5.6.3.1a)

$V_{ci} = 0.6(0.95)\sqrt{8000}\,(5 \times 21.5) + 1146 + \left(\frac{21980 \times 698050}{826900}\right)$

$V_{ci} = \underline{25.18\ Kips} \longleftarrow$ control (V_{ci})

ACI 22.5.6.3.1 b&c

$V_i = V_u - V_d$

$V_i = 1.85(12.5) - 1146$

$V_i = 21.98 \approx \underline{22\ kip}$

$M_{max} = M_u - M_d$

$= 75 - 6.11$

$M_{max} \approx 68.9\ k\text{-}ft$

$M_{max} = \underline{826.9\ kip\text{-}in}$

(b) $A_{ps} \cdot f_{se} < \underbrace{(2)}_{2\ strands}(0.153\ in^2)(174) = \underline{53\ kip} > 31.8\ kip$

$0.4 \cdot A_{ps} \cdot f_{pu} = 0.4(2)(0.153)\underbrace{(260)}_{ultimate} ksi = \underline{31.8\ kip}$

(c) $V_{ci} = 2\lambda\sqrt{f'_c} \cdot b_w \cdot d$

$= 2(0.95)\sqrt{6000}\,(5'')(21.5') = \underline{\mathbf{18.3\ kip}}$

(3) $V_{cw} = (3.5 \cdot \lambda \cdot \sqrt{f'_c} + 0.3 f_{pc}) b_w \cdot d_p$

$V_{cw} = \left(3.5 \times 0.95 \times \sqrt{6000} + 0.3(53000)\right) \times \frac{5 \times 21.5}{1000}$

$V_{cw} = \underline{1741\ kips}$

(Compressive stress at centroid) $\cdot\ f_{pc} = \frac{P}{A}$

$= \frac{(174)(2)(0.153)}{232\ in^2}$

$= \underline{53\ Ksi}$

Required $A_{v,min}$ (ACI 9.6.3.4)

Prestressed with $A_{ps}f_{se} \geq 0.4\left(A_{ps}f_{pu} + A_s \cdot f_y\right)$

Lesser

* $0.75\sqrt{f'_c}\cdot\dfrac{b_w}{f_{yt}} = 0.75\sqrt{5000}\cdot\dfrac{5}{60000} = 5.59\times10^{-3}$

* $50\,\dfrac{b_w}{f_{yt}} = 50\times\dfrac{5}{60000} = 4.16\times10^{-3}$

Greater

* $\dfrac{A_{ps}\,f_{pu}}{80\,f_y\cdot d}\sqrt{\dfrac{d}{b_w}} = \dfrac{2(0.153)(265)}{80\times60000\times21.5}\sqrt{\dfrac{21.5}{5}} = 1.63\times10^{-3}$

=

Spacing maximum

[ACI 9.7.6.2.2]

$\dfrac{3h}{4} = 0.75\times24$

$= 18''$

$\dfrac{A_{v_{min}}}{s} = 1.63\times10^{-3}\ in^2/in$

$A_{v\,min} = 1.63\times10^{-3}\times18 = 0.02934\ in^2$

PROVIDE No.3 @ 18" oc

~~$A_{v\,min} = 0.02934\times264 = 7.75\ in^2$~~

$(0.11in^2 > 0.03in^2)$

$\dfrac{A_v}{s} = \dfrac{0.11in^2}{18} = 0.0061 > 0.0016$

Minimum Shear provided

$$V_s = \frac{A_v \cdot f_y \cdot d}{S} = \frac{(\cancel{0.022})^{\,0.11in^2} \times (60ksi) \times (21.5)}{18''} = \cancel{24.022 \text{ kips}} \;\; 7.9 \text{ kip}$$

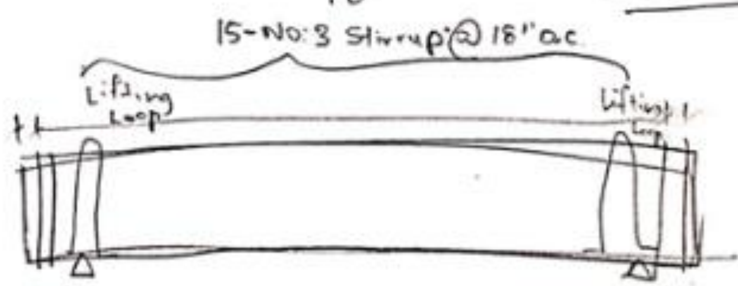
Try 4@ No:3

$$V_s = \frac{0.44 \times 60 \times 21.5}{18} = 31.53 \text{ kips} > 25.18 \text{ kip}$$

with 18" spacing

Also we can try 2 #4

$$V_s = \frac{0.40 \times 60 \times 21.5}{18} = 28.\overline{6} \text{ kips} > 25.18 \text{ kips}$$



$V_c = 25.2^{kip}$

$\phi V_n = 0.75(25.2^k + 7.9^k) = 24.8^{kip}$

$$\frac{24.8^k}{11.9} = \underline{2.08}$$

Appendix L – Table of Decision Matrix

Beam Decision Matrix							
Cross Section	Cost ($)	Score	*Weight of Beam (pcf)*	Score	Deflection (inches)	Score	Total Score
T Beam LW	218.52	1.48	*6069.18*	5.17	7.04	10.00	16.65
T Beam NW	218.52	1.48	*7277.99*	0.91	7.25	5.96	8.35
I Beam LW	**183.13**	**10.00**	***4697.05***	**10.00**	**7.25**	**6.09**	**26.09**
I Beam NW	183.13	10.00	*5632.58*	6.71	7.46	1.97	18.68
Hollow Box Beam LW	224.66	0.00	*6285.22*	4.41	7.35	4.06	8.47
Hollow Box Beam NW	224.66	0.00	*7537.07*	0.00	7.57	0.00	0.00

Appendix M – Tables of Time Breakdown

Project Time Estimate Breakdown Prediction						
Task	SENG Hours	ENG Hours	LAB Hours	INT Hours	AA Hours	Total Hours per Task
Task 1: Research	14	39	0	53	12	118
Task 2: Preliminary Design	1	16	16	20	0	53
Task 3: Final Design and Analysis	2	10	0	20	0	32
Task 4: Predictions	1	12	0	16	3	32
Task 5: Shop Drawings	2	11	8	22	5	48
Task 6: Casting of Beam	0	11	0	15	0	26
Task 7: Testing of Beam	0	9	6	12	0	27
Task 8: Project Management	33	104	39	35	21	232
Total Hours	53	212	69	193	41	568

Project Time Estimate Breakdown Actual						
Task	SENG Hours	ENG Hours	LAB Hours	INT Hours	AA Hours	Total Hours per Task
Task 1: Research	5	11	5	13	2	36
Task 2: Preliminary Design	5	37	0	56	3	101
Task 3: Final Design and Analysis	14	21	0	73	0	108
Task 4: Predictions	2	7	3	13	4	29
Task 5: Shop Drawings	5	30	0	34	1	70
Task 6: Casting of Beam	0	0	0	0	0	0
Task 7: Testing of Beam	0	0	3	0	0	3
Task 8: Project Management	43	91	5	26	18	183
Total Hours	74	197	16	215	28	530

Appendix N – Tables of Cost

Cost of Project Hours Prediction			
Job Title	Billing Rate ($/hour)	Hours Worked	Total Cost ($)
SENG	257	53	13621
ENG	132	212	27984
LAB	44	69	3036
INT	28	193	5404
AA	33	41	1353
Total			$ 51,398

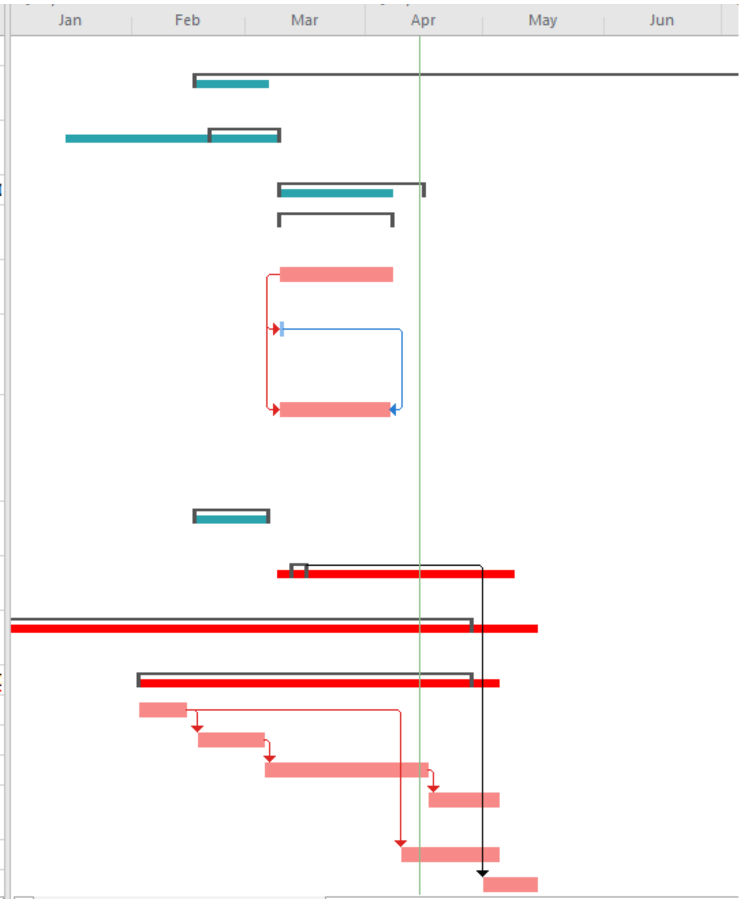
Cost of Project Hours Actual			
Job Title	Billing Rate ($/hour)	Hours Worked	Total Cost ($)
SENG	257	74	19018
ENG	132	197	26004
LAB	44	16	704
INT	28	215	6020
AA	33	28	924
Total			$ 52,670

Cost of Materials				
Material	Cost per Unit ($/unit)	Unit	Amount of Material	Total Material Cost ($)
Concrete	100	yd^3	1.27	127
Stirrups	1.045	lb	15	15.675
Formwork	1.25	ft^2	205.48	256.85
Strands	0.6	LF	22	13.2
Compression Steel	0.9045	LF	22	19.899
Total				$ 432.62

Cost of Labor	
Cost	$ 675

Appendix O – Figure of Updated Schedule

Task Name	Duration	Start	Finish
Task 1: Research	**43 days?**	**Mon 9/9/19**	**Tue 11/5/19**
Task 2: Preliminary Beam Design	**114 days?**	**Mon 2/17/20**	**Thu 7/23/20**
Task 3: Final Design and Analysis	**12 days?**	**Fri 2/21/20**	**Mon 3/9/20**
Task 4: Predictions	**27 days?**	**Tue 3/10/20**	**Wed 4/15/2(**
Task 4.1: Response 2000	21 days?	Tue 3/10/20	Tue 4/7/20
Task 4.1.1: Cracking Load	21 days?	Tue 3/10/20	Tue 4/7/20
Task 4.1.2: Max Load at Midspan	1 day?	Tue 3/10/20	Tue 3/10/20
Task 4.1.3: Maximum Anticipated Deflection	20 days?	Tue 3/10/20	Mon 4/6/20
Task 5: Shop Drawings	**15 days?**	**Mon 2/17/20**	**Fri 3/6/20**
Task 6: Casting of Beam	**2 days?**	**Fri 3/13/20**	**Mon 3/16/20**
Task 8: Project Management	**163 days?**	**Fri 9/13/19**	**Mon 4/27/20**
Task 8.1 Report	61 days?	Mon 2/3/20	Mon 4/27/2(
Task 8.1.1: 30%	10 days	Mon 2/3/20	Fri 2/14/20
Task 8.1.2: 60%	13 days?	Tue 2/18/20	Thu 3/5/20
Task 8.1.3: 90%	30 days?	Fri 3/6/20	Thu 4/16/20
Task 8.1.4: Final Report	12 days?	Fri 4/17/20	Mon 5/4/20
Task 8.2: Website	17 days?	Fri 4/10/20	Mon 5/4/20
Task 8.3: Video	10 days?	Fri 5/1/20	Thu 5/14/20



Appendix P – Tables of Loss Calculations

Elastic Shortening Losses		
K_{es}	1	
E_{ps} (psi)	2.85E+07	
E_{ci} (psi)	4031000	MathCAD
f_{cir} (psi)	4.78E+02	
	K_{cir}	0.9
	P_i (lb)	59670
MathCAD	A_g (in^2)	227.5
	e (in)	9.875
	I_g (in^4)	1.70E+04
	M_g (in.lb)	114600
ES (psi)	3379.32	

Losses Due to Creep of Concrete		
K_{cr}	1.6	L-W Concrete
E_{ps} (psi)	2.85E+07	
E_c (psi)	5098000	MathCAD
f_{cir} (psi)	4.78E+02	
f_{cds} (psi)	0	
	CR (psi)	4275.26

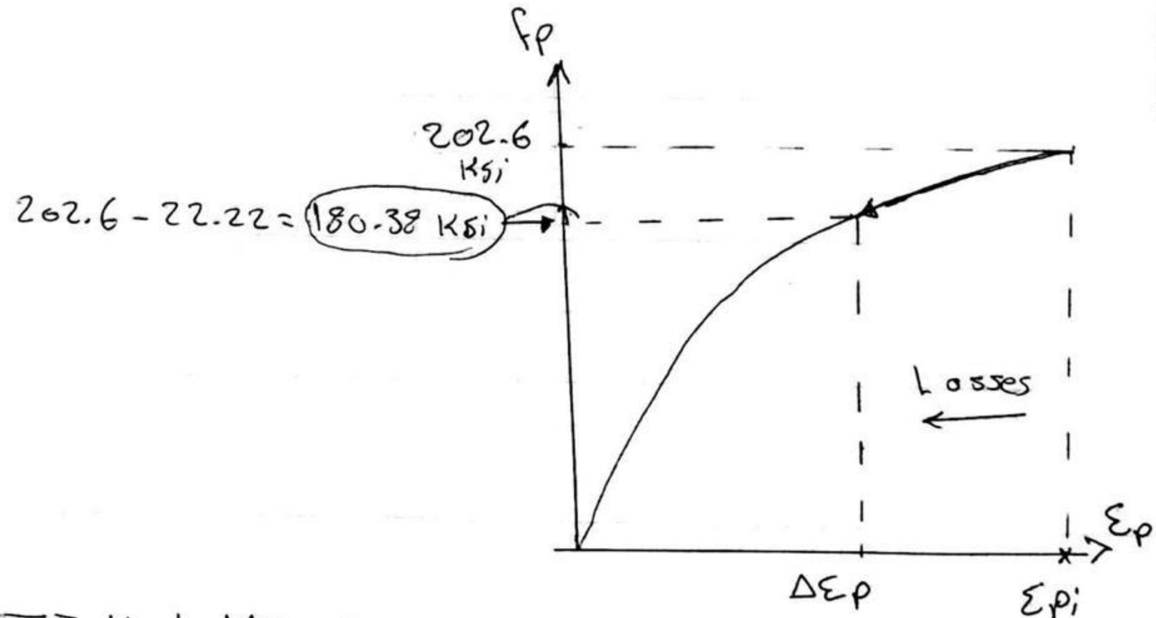
Losses Due to Shrinkage of Concrete		
K_{sh}	1	
E_{ps} (psi)	2.85E+07	
V (in^2)	227.5	
S	114.078	
V/S (in)	1.994249549	
RH %	50	Assumed for Az
	SH (psi)	10286.83

Losses Due to Relaxation		
K_{re}	5000	Table 5.7.1
J	0.04	
SH (psi)	10286.832	
CR (psi)	4.28E+03	
ES (psi)	3.38E+03	
f_{pi}/f_{pu}	0.7222222	
C	1	Table 5.7.2
	RE (psi)	4282.34

Total Losses (psi)	2.22E+04
Total Losses (ksi)	22.22

Appendix Q – Figure of Hand Calculations for Loss of Strain

Total Losses = 22.22 Ksi



=> Modulus of Elasticity of strand (E) = 28500 ksi

$$E = \frac{\sigma}{\epsilon},$$

$$\Rightarrow \varepsilon_{p_i} = \frac{\sigma}{E} = \frac{202.6\ \text{ksi}}{28500\ \text{ksi}} = \boxed{0.00711}$$

$$\Rightarrow \Delta\varepsilon_p = \frac{180.38\ \text{Ksi}}{28500\ \text{Ksi}} = \boxed{0.00633}$$

Scanned with CamScanner

Appendix R – Figure of Hand Calculations for Deflection

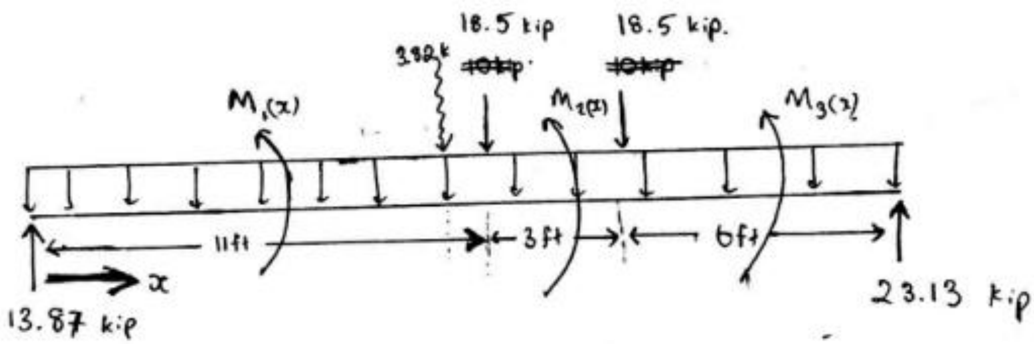
$\boxed{\text{Deflection \& Camber Calculations}}$

* Slope, $\theta = \frac{dy}{dx}$ / * curvature $= \frac{1}{\rho} = \frac{M}{EI}$. / Deflection, $\Delta = y(x)$

$$\boxed{\theta_{(x)} = \int \frac{M}{EI} \cdot dx} \qquad \boxed{\Delta_{(x)} = \int \theta \cdot dx = \iint \frac{M}{EI} \cdot dx}$$

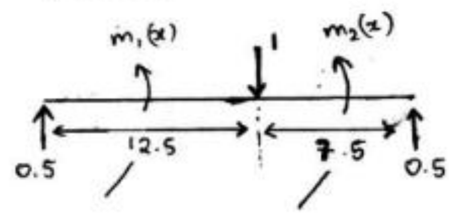
Deflection Analysis from virtual Work Method.

Real



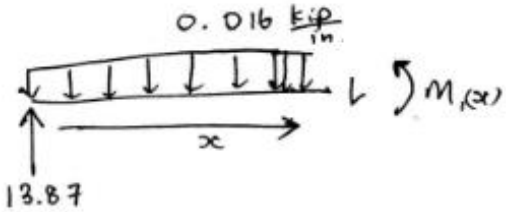
Distributed dead load = 0.191 kip/ft
= 0.016 kip/in

virtual



⊛ Determining $M(x)$ & $m(x)$ from section cuts

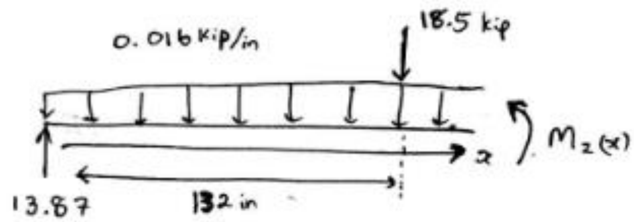
$M_1(x)$



$$m_1(x) + (0.016 \times x) \times \frac{x}{2} - (13.87\,x) = 0$$

$$\boxed{m_1(x) = 13.87x - 0.008\,x^2}$$

$m_2(x)$

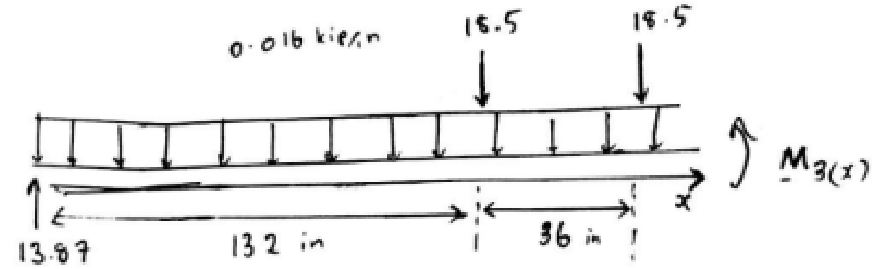


$$m_2(x) + 18.5(x-132) + (0.016 \times x) \times \frac{x}{2} - 13.87(x) = 0$$

$$M_2(x) = 13.87x - 18.5x + 2442 - 0.008x^2$$

$$\boxed{M_2(x) = -4.63x - 0.008x^2 + 2442}$$

$M_{3}(x)$



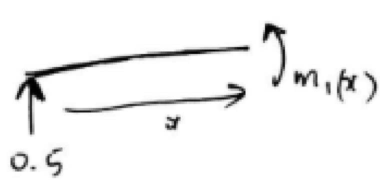
$$M_{(3)(x)} + 18.5(x-168) + 18.5(x-132) - 13.87(x) + (0.016 \times x) \times x/2 = 0$$

$$M_{3(x)} = 13.87(x) - 18.5x + 3108 - 18.5x + 2442 - 0.008x^2$$

$$\boxed{M_{3(x)} = -23.13x - 0.008x^2 + 5550}$$

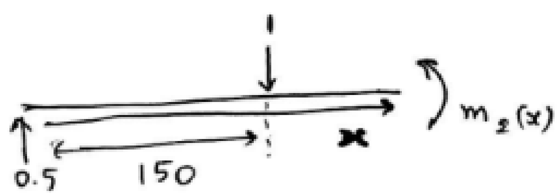
For virtual work

$m_1(x)$



$$\boxed{m_{1}(x) = 0.5(x)}$$

$m_2(x)$



$$m_2(x) + 1(x-150) - 0.5(x) = 0$$

$$m_2(x) = 0.5(x) - x + 150$$

$$\boxed{m_2(x) = -0.5x + 150}$$

Deflection

$$* \; EI \cdot \Delta = \int_0^{132} m_1 M_1(x) + \int_{132}^{150} m_1(x) M_2(x) + \int_{150}^{168} m_2(x) M_2(x)$$

$$+ \int_{168}^{240} m_2(x) \; M_3(x)$$

$$\Delta_{(D+L)} \downarrow \text{ (at ultimate)} = \underline{\underline{\qquad}}$$

(*) solve for camber.

$$\uparrow \Delta_c = \frac{1}{8} \frac{P \cdot e L^2}{E I_e}$$

$$= \frac{1}{8} \cdot \frac{53.244 \times 9.875 \times 240^2}{5098 \times 17681.5}$$

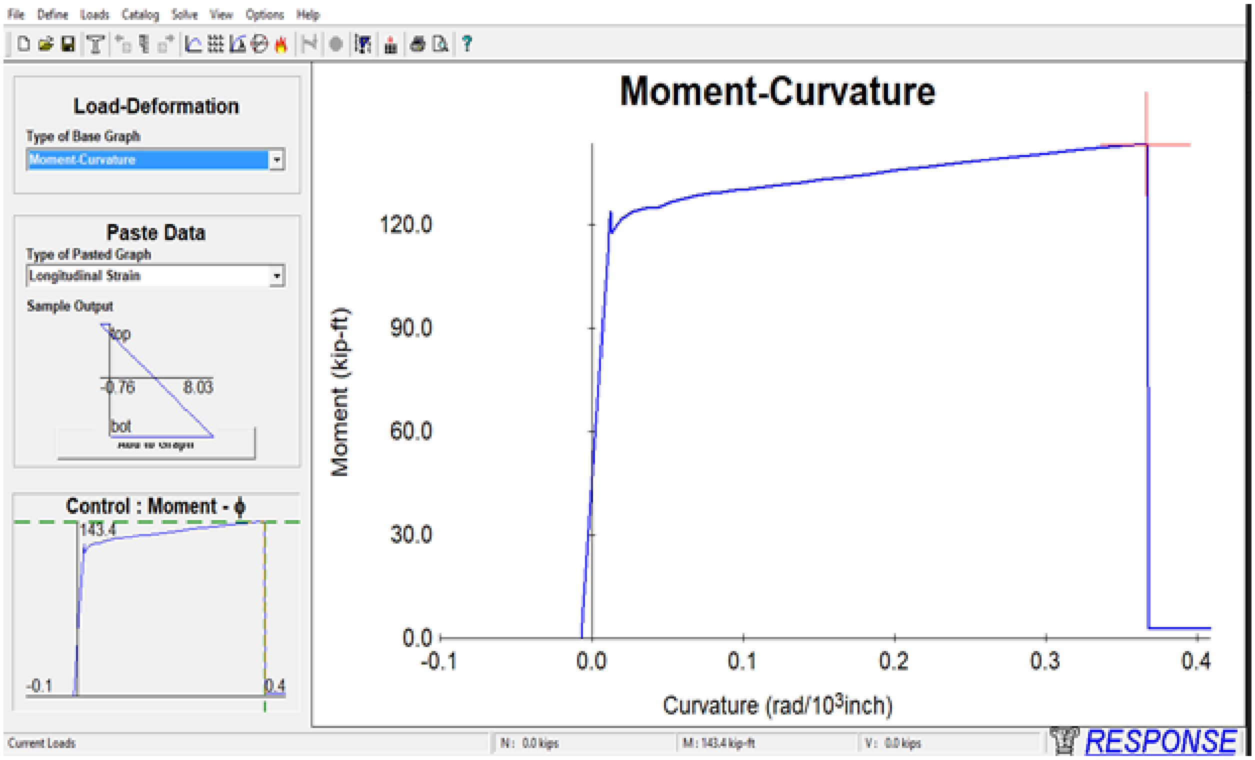
$$= \underline{\underline{0.042 \text{ in}}}$$

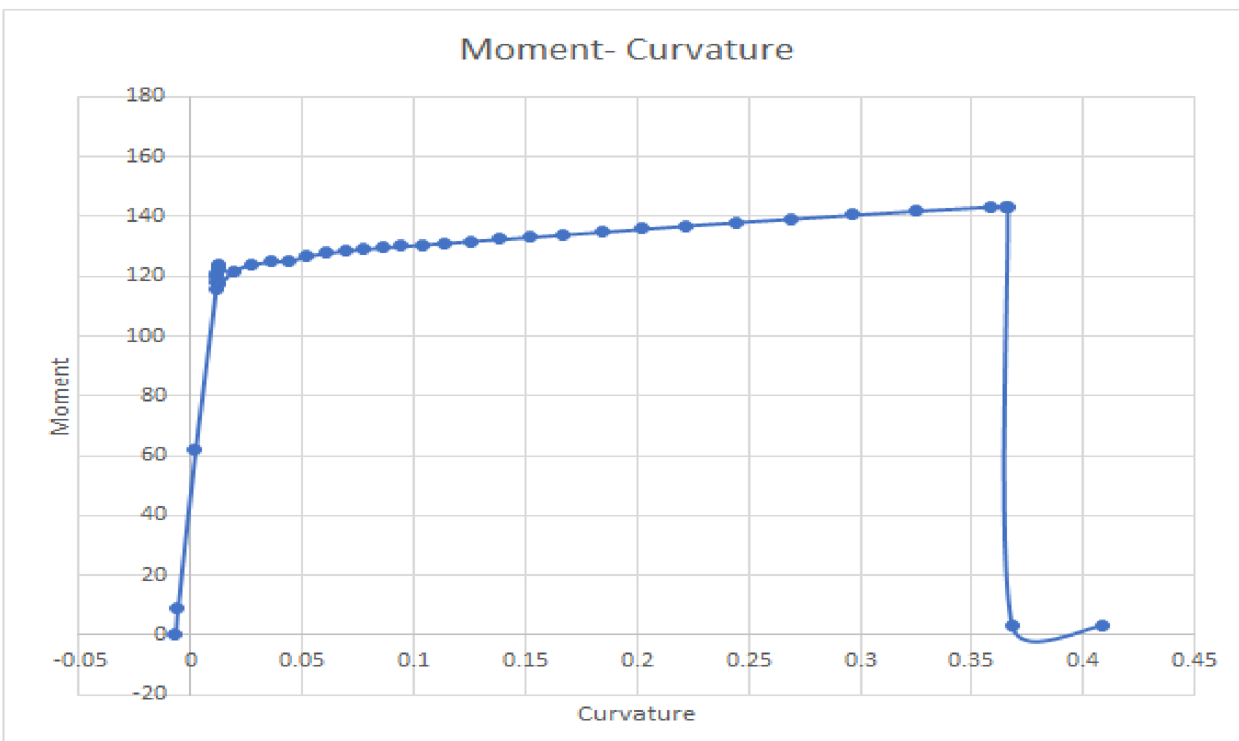
* ∴ Total Predicted deflection could be;

$$\underline{\underline{\Delta_{D+L}(\downarrow) + \Delta_{camber}(\uparrow)}}$$

Appendix S – Table of Raw Data from Response 2000 Moment Curvature Data

Curvature (rad/10^3inch)	Moment (kip-ft)	Curvature (rad/inch)
Line type : 0		
-0.007	0.008	-0.000007
-0.006	8.912	-0.000006
0.002	62.329	0.000002
0.011	115.74	0.000011
0.011	117.877	0.000011
0.011	119.586	0.000011
0.011	120.953	0.000011
0.012	122.047	0.000012
0.012	122.922	0.000012
0.012	123.622	0.000012
0.012	124.182	0.000012
0.012	117.542	0.000012
0.019	121.843	0.000019
0.027	124.024	0.000027
0.036	124.864	0.000036
0.044	125.277	0.000044
0.052	126.753	0.000052
0.061	127.848	0.000061
0.069	128.575	0.000069
0.077	129.142	0.000077
0.086	129.639	0.000086
0.094	130.113	0.000094
0.104	130.619	0.000104
0.114	131.171	0.000114
0.125	131.778	0.000125
0.138	132.451	0.000138
0.152	133.199	0.000152
0.167	134.025	0.000167
0.184	134.937	0.000184
0.202	135.947	0.000202
0.222	137.064	0.000222
0.244	138.181	0.000244
0.269	139.388	0.000269
0.296	140.664	0.000296
0.325	142.018	0.000325
0.358	143.327	0.000358
0.365	143.384	0.000365
0.366	143.376	0.000366
0.368	2.822	0.000368
0.409	2.876	0.000409


File Define Loads Catalog Solve View Options Help
Load-Deformation
Type of Base Graph
Moment-Curvature
Paste Data
Type of Pasted Graph
Longitudinal Strain
Sample Output
top
-0.76
8.03
bot
Control : Moment - ϕ
143.4
-0.1
0.4
Moment-Curvature
Moment (kip-ft)
120.0
90.0
60.0
30.0
0.0
-0.1
0.0
0.1
0.2
0.3
0.4
Curvature (rad/10³inch)
Current Loads
N: 0.0 kips
M: 143.4 kip-ft
V: 0.0 kips
RESPONSE


Moment- Curvature
Moment
180
160
140
120
100
80
60
40
20
0
-20
-0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
Curvature

Appendix T – Table of Calculations in Excel

x	x	M(x)	M(x)	m(x)	dx	M/EI (x)	(m.M/EI)	Integrate(m.M/EI).dx
[in]	[ft]	[k*in]	[Kip*ft]	[k*in]		[rad/in]		[in]
0	0.00	0.0	0.00	0	4			0
4	0.33	55.4	4.61	2	4	-6.48E-06	-1.297E-05	0.0000
8	0.67	110.4	9.20	4	4	-5.97E-06	-2.387E-05	-0.0001
12	1.00	165.3	13.77	6	4	-5.45E-06	-3.272E-05	-0.0002
16	1.33	219.9	18.32	8	4	-4.94E-06	-3.954E-05	-0.0003
20	1.67	274.2	22.85	10	4	-4.43E-06	-4.435E-05	-0.0005
24	2.00	328.3	27.36	12	4	-3.93E-06	-4.714E-05	-0.0007
28	2.33	382.1	31.84	14	4	-3.42E-06	-4.795E-05	-0.0009
32	2.67	435.6	36.30	16	4	-2.92E-06	-4.678E-05	-0.0011
36	3.00	489.0	40.75	18	4	-2.42E-06	-4.365E-05	-0.0012
40	3.33	542.0	45.17	20	4	-1.93E-06	-3.857E-05	-0.0014
44	3.67	594.8	49.57	22	4	-1.43E-06	-3.155E-05	-0.0015
48	4.00	647.3	53.94	24	4	-9.42E-07	-2.262E-05	-0.0017
52	4.33	699.6	58.30	26	4	-4.53E-07	-1.178E-05	-0.0017
56	4.67	751.6	62.64	28	4	2.05E-06	5.729E-05	-0.0016
60	5.00	803.4	66.95	30	4	2.69E-06	8.076E-05	-0.0014
64	5.33	854.9	71.24	32	4	3.33E-06	1.067E-04	-0.0010
68	5.67	906.2	75.51	34	4	3.97E-06	1.351E-04	-0.0005
72	6.00	957.2	79.76	36	4	4.61E-06	1.660E-04	0.0001
76	6.33	1007.9	83.99	38	4	5.24E-06	1.993E-04	0.0008
80	6.67	1058.4	88.20	40	4	5.87E-06	2.350E-04	0.0017
84	7.00	1108.6	92.39	42	4	6.50E-06	2.731E-04	0.0027
88	7.33	1158.6	96.55	44	4	7.13E-06	3.135E-04	0.0039
92	7.67	1208.3	100.69	46	4	7.75E-06	3.563E-04	0.0052
96	8.00	1257.8	104.82	48	4	8.36E-06	4.014E-04	0.0067
100	8.33	1307.0	108.92	50	4	8.98E-06	4.489E-04	0.0084
104	8.67	1356.0	113.00	52	4	1.95E-05	1.016E-03	0.0114
108	9.00	1404.6	117.05	54	4	1.10E-05	5.940E-04	0.0146
112	9.33	1453.1	121.09	56	4	1.10E-05	6.160E-04	0.0170
116	9.67	1501.3	125.11	58	4	3.86E-05	2.238E-03	0.0227
120	10.00	1549.2	129.10	60	4	7.48E-05	4.487E-03	0.0362
124	10.33	1596.9	133.07	62	4	1.50E-04	9.301E-03	0.0638
128	10.67	1644.3	137.02	64	4	2.21E-04	1.416E-02	0.1107
132	11.00	1691.4	140.95	66	4	3.02E-04	1.994E-02	0.1789
136	11.33	1664.4	138.70	68	4	2.54E-04	1.728E-02	0.2533
140	11.67	1637.0	136.42	70	4	2.10E-04	1.473E-02	0.3173
144	12.00	1609.4	134.12	72	4	1.69E-04	1.214E-02	0.3711
148	12.33	1581.5	131.79	74	4	1.25E-04	9.271E-03	0.4139
152	12.67	1553.4	129.45	76	4	8.14E-05	6.183E-03	0.4448
						Deflection without Cam		2.27
						Total Deflection with Ca		2.23

Curved Region	
Moments	Interpolated Curvature Values
100.69	7.74574E-06
104.82	8.36307E-06
108.92	8.9772E-06
113.00	1.95367E-05
117.05	1.12214E-05
121.09	0.000011
125.11	3.85929E-05
129.10	7.47772E-05
133.07	0.000150008
137.02	0.000221194
140.95	0.000302136
138.70	0.000254143
136.42	0.00021037
134.12	0.000168653
131.79	0.00012529
129.45	8.13551E-05

Linear Region	
Moments	Interpolated Curvature values
4.61	-6.48285E-06
9.20	-5.96721E-06
13.77	-5.45395E-06
18.32	-4.9431E-06
22.85	-4.43464E-06
27.36	-3.92857E-06
31.84	-3.4249E-06
36.30	-2.92363E-06
40.75	-2.42475E-06
45.17	-1.92827E-06
49.57	-1.43419E-06
53.94	-9.42498E-07
58.30	-4.53205E-07
62.64	2.04598E-06
66.95	2.69206E-06
71.24	3.33496E-06
75.51	3.97465E-06
79.76	4.61115E-06
83.99	5.24446E-06
88.20	5.87457E-06
92.39	6.50149E-06
96.55	7.12521E-06